Why a Background Persistent Field Must Exist in the Extended Theory of General Relativity

Patrick Marquet*

Abstract: In the framework of an Extended General Relativity based on a semi-affine connection, we have postulated the existence of a background persistent field filling the physical vacuum and affecting the neighboring masses. In a holonomic scheme, the original Weyl formulation for generalized variational fields leads to the energy-momentum tensor of a perfect fluid in the Einstein field equation with a massive source. Since both the Weyl and EGR connections are shown to be equivalent in a particular way, the perfect fluid tensor with its pressure appears as a Riemannian transcription of the EGR massive tensor, with its surrounding active background field. This result would lend support to our assertion regarding the persistent field within the EGR formulation.

Contents:

Introduction 79

Chapter 1 The Extended General Relativity (EGR)

§1.1 The Riemannian metric 80

§1.2 Brief overview on the Extended General Relativity 80

§1.2.1 Basic properties 80

§1.2.2 The EGR world-velocity 81

§1.3 The EGR persistent field 82

Chapter 2 The Perfect Fluid Solution

§2.1 The Weyl formulation 82

§2.2 The holonomic scheme 83

§2.3 The fiber bundle framework 83

§2.3.1 Short overview on fiber bundles 83

§2.3.2 Variational calculation 83

§2.3.3 Specific variational derivation 85

Chapter 3 The Geodesic Equations

§3.1 The Riemannian situation 86

§3.2 The EGR geodesics for the EGR massive tensor 88

*Postal address: 7, rue du 11 nov, 94350 Villiers/Marne, Paris, France. E-mail: patrick.marquet6@wanadoo.fr. Tel: (33) 1-49-30-33-42.

§3.2.1 A conformal-like 4-velocity 88
§3.2.2 Relation between the Weyl and the EGR connections 88
§3.2.3 The EGR geodesic equation 89
Consequences and conclusions 89

Introduction. In one of our earlier publications [1], we have worked out an extended theory of general Relativity (EGR Theory) which allows for a permanent field to exist, thus filling the physical vacuum. This field appears as a continuity of the matter-pseudo-gravity field [2] which is required to fulfill the conservation law for the corresponding energy-momentum tensor in the classical GR theory. The existence of this persistent field has been first predicted in another paper based on the Lichnerowicz matching conditions applied to two spherically symmetric metrics [3]. In the foregoing, we provide a strict demonstration based on the properties of the so-called *generalized variational manifolds* [4, 5], which are a general class of *Finslerian spaces.* This theory relies on the symmetric *Weyl connection* which can be extended to the so-called *decomposable connection* or *semi-symmetric connection* [6, p. 69–75]. The Weyl manifold is then entirely defined from a) the Riemannian metric ds^2 and b) a form $dK = K_a\, dx^a$ which is generally non-integrable.

In what follows, we will however restrict our study to the symmetric part of the Weyl connection which readily relate to the EGR one in a very simple way.

With respect to a holonomic frame (in the sense of Cartan), the form dK becomes integrable, and we may establish a *pure* conformal metric $(ds^2)'$ whose conformal factor is e^{2K}.

This conformal factor enables us to define 4-velocities collinear with the unit 4-vectors of the Einstein metric and allow us to write simple conformal geodesic equations for the flow lines of a specific type of fluid. This differential system is nothing else but the geodesic equation of a *perfect fluid*, where an equation of state links its proper density and the pressure prescribed on it: $\rho = f(p)$. So, by choosing a Weyl connection that spans the *generalized variational spaces* and making use of a holonomic frame, we are led to find the energy-momentum tensor of a neutral perfect fluid T_{ab}.

By relating the Weyl connection to the EGR connection in a very simple way, the EGR massive tensor $(T_{ab})_{\mathrm{EGR}}$ appears to be formally equivalent to the form of T_{ab}.

This substantiates the existence of the EGR persistent field tensor which in the Riemannian scheme is represented by a pressure term $g_{ab}\, p$, and where the dynamical mass density ρ increases to $(\rho+p)$, thus confirming our postulate that the EGR Theory describes trajectories of dynamical entities comprising *bare* masses of particles and their own gravity field [7].

Chapter 1. The Extended General Relativity (EGR)

§1.1. The Riemannian metric. In an open neighborhood of the pseudo-Riemannian manifold V_4, the metric of signature $(+---)$ can be expressed by

$$ds^2 = g_{ab}\,\theta^a\theta^b, \tag{1.1}$$

where θ_a are the Pffafian forms in the considered region $a=4,1,2,3$.

The manifold considered here is always understood to be *globally hyperbolic* [8]. We also set here $c=1$.

§1.2. Brief overview on the Extended General Relativity

§1.2.1. Basic properties. We first briefly recall here our previous results. The non-metricity condition is ensured by the EGR covariant derivative D or $'$, of the metric tensor which has been found to be

$$\mathrm{D}_a\, g_{bc} = \frac{1}{3}\big(J_c\, g_{ab} + J_b\, g_{ac} - J_a\, g_{bc}\big). \tag{1.2}$$

The vector J_a is related to a specific 4×4 *Hermitean* $(\gamma^5)_{\text{EGR}}$ matrix by

$$J^a = k\ \mathrm{tr}\,(\gamma^5)_{\text{EGR}}\,, \tag{1.3}$$

where k is a real positive constant [9].

One then considers the *semi-affine connection* (EGR connection)

$$(\Gamma^d_{ab})_{\text{EGR}} = \{^{\,d}_{ab}\} + (\Gamma^d_{ab})_J \tag{1.4}$$

with

$$(\Gamma^d_{ab})_J = \frac{1}{6}\big(\delta^d_a\, J_b + \delta^d_b\, J_a - 3\, g_{ab}\, J_d\big) \tag{1.5}$$

and the Christoffel symbols of the second kind $\{^{\,d}_{ab}\}$.

If ∇_a is the Riemannian derivative operator, we have thus inferred the EGR curvature tensor:

$$(R^a_{\cdot\, bcd})_{\text{EGR}} = R^a_{\cdot\, bcd} + \nabla_d\,\Gamma^a_{bc} - \nabla_c\,\Gamma^a_{bd} + \Gamma^f_{bc}\,\Gamma^a_{fd} - \Gamma^f_{bd}\,\Gamma^a_{fc}\,. \tag{1.6}$$

The contracted tensor

$$\left.\begin{aligned} (R_{ab})_{\mathrm{EGR}} &= R_{ab} - \frac{1}{2}\left(g_{ab}\,\nabla_e\,J^e + \frac{1}{3}\,J_a\,J_b\right) + \frac{1}{6}\,J_{ab} \\ J_{ab} &= \partial_a\,J_b - \partial_b\,J_a \end{aligned}\right\} \tag{1.7}$$

leads to the EGR *Einstein tensor*

$$(G_{ab})_{\mathrm{EGR}} = (R_{ab})_{\mathrm{EGR}} - \frac{1}{2}\left(g_{ab}\,R_{\mathrm{EGR}} - \frac{2}{3}\,J_{ab}\right) \tag{1.8}$$

with the EGR *curvature scalar*

$$R_{\mathrm{EGR}} = R - \frac{1}{3}\left(\nabla_e\,J^e + \frac{1}{2}\,J^2\right). \tag{1.9}$$

In the *Riemannian regime*, this tensor obviously reduces to the usual Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2}\,g_{ab}\,R\,.$$

§1.2.2. The EGR world-velocity. On the EGR manifold $\mathfrak{M}$, the conoids, as defined in the Riemannian scheme, do not exactly coincide with the EGR representation, because the EGR line element slightly deviates from the standard Einstein geodesic invariant [10].

The EGR line element includes a small correction to the Riemann invariant ds^2 which we write as

$$(ds^2)_{\mathrm{EGR}} = ds^2 + d\,(ds^2)\,, \tag{1.10}$$

where

$$d\,(ds^2) = (\mathrm{D}\,g_{ab})\,dx^a dx^b \tag{1.11}$$

with $\mathrm{D}\,g_{ab} = \frac{1}{3}\,(J_c\,g_{ab} + J_b\,g_{ac} - J_a\,g_{bc})\,dx^c$.

Hence, the EGR line element is simply expressed by

$$(ds^2)_{\mathrm{EGR}} = \big(g_{ab} + \mathrm{D}\,g_{ab}\big)\,dx^a dx^b, \tag{1.12}$$

which naturally reduces to the Riemannian (invariant) interval ds^2 when the covariant derivative of the metric tensor g_{ab} vanishes (i.e. when we have $J_a = 0$).

We can thus define an EGR 4-*velocity* as

$$(u^a)_{\mathrm{EGR}} = \frac{dx^a}{(ds)_{\mathrm{EGR}}}\,. \tag{1.13}$$

This vector will always be assumed to be a unit vector according to

$$g_{ab}\,(u^a u^b)_{\text{EGR}} = g^{ab}\,(u_a u_b)_{\text{EGR}} = 1\,. \tag{1.14}$$

§1.3. The EGR persistent field. The EGR field equation with a massive source is written

$$(G_{ab})_{\text{EGR}} = \varkappa\left[\,\rho_{\text{EGR}}(u_a u_b)_{\text{EGR}} + (t_{ab})_{\text{field}}\right], \tag{1.15}$$

where $\varkappa$ is Einstein's constant.

The persistent field tensor is here assumed to represent a *vacuum homogeneous background energy* which is linked to its density by

$$\sqrt{-g}\,(t^{ab})_{\text{field}} = (\mathcal{F}^{ab})_{\text{field}}\,. \tag{1.16}$$

Explicitly, $(\mathcal{F}^{ab})_{\text{field}}$ is derived from the canonical equations

$$(\mathcal{F}^a_b)_{\text{field}} = \frac{1}{2\varkappa}\left[\mathcal{H}\,\delta^a_b - \partial_b(\Gamma^e_{df})_{\text{EGR}}\,\frac{\partial\mathcal{H}}{\partial(\partial_a\Gamma^e_{df})_{\text{EGR}}}\right], \tag{1.17}$$

where the invariant density is $\mathcal{H} = (\mathcal{R}^{ab}R_{ab})_{\text{EGR}}$ built itself with the second-rank tensor density

$$(\mathcal{R}^{ab})_{\text{EGR}} = (R^{ab})_{\text{EGR}}\,\sqrt{-g}\,.$$

Chapter 2. The Perfect Fluid Solution

§2.1. The Weyl formulation. The essential work of Lichnérowicz on *The Generalized Variational Spaces* begins by defining the *symmetric Weyl connection*:

$$W^a_{bc} = \left\{{a \atop bc}\right\} + g^{ad}\left(g_{cd}\,F_b + g_{bd}\,F_c - g_{bc}\,F_d\right). \tag{2.1}$$

From the point M in the neighborhood of the space-time manifold V_4, a congruence of differentiable lines, such that for all $m \in \mathrm{V}_4$ there is a unique curve joining M onto m, and thus the *Weyl metric*

$$(ds^2)_W = e^{2F} ds^2 \tag{2.2}$$

can be defined, where

$$F = \int_{\mathrm{M}}^{m} K_a\,dx^a. \tag{2.3}$$

The form $dK = K_a\,dx^a$ is generally *non-integrable*.

§2.2. The holonomic scheme. In a *holonomic frame* [11, p. 45], we have $K_a = \partial_a K$, and the form dK is now integrable. In this case, we simply write (2.2) as the conformal metric

$$(ds^2)' = e^{2K} ds^2 \tag{2.4}$$

and the Weyl connection (2.1) reduces to the conformal connection

$$\{^{\,a}_{bc}\}' = \{^{\,a}_{bc}\} + g^{ad}\big(g_{cd} K_b + g_{bd} K_c - g_{bc} K_d\big). \tag{2.5}$$

§2.3. The fiber bundle framework

§2.3.1. Short overview on fiber bundles. Given an n-dimensional manifold M, we can construct another manifold called a *fiber bundle* which is locally a direct product of M and a suitable space E, called the *total space*. For a thorough theory see for example [12, p. 50–55]. In this section, we shall only consider the *tangent bundles* category T_p(M), ($p \in$ M) which is the fiber bundle over the manifold M obtained by giving the set $\mathcal{E} = \cup\, T_p$, its natural manifold structure and its natural projection onto M. A trivial example is the manifold M representing the circle S_1 and the real line R_1 with which can be constructed the cylinder C_2, as a product bundle over S_1.

In the following we shall consider:

- The *differentiable manifold* V_4;
- The *bundle fiber space* $W_{2\times 4}$ of all vectors tangent to various points of V_4;
- The *bundle fiber space* D_{8-1} of all directions tangent to various points of V_4.

§2.3.2. Variational calculation. Most of the derivations detailed in here can be found in [13, p. 72–75]. An element $\in W_8$ is defined by the coordinates x^a of the point $x \in V_4$, and by the four quantities ${}^\circ x^a$, contravariant components of the vector in the natural basis associated at x to the (x^a). For an element of D_{8-1}, the ${}^\circ x^a$ will be only defined as *directional parameters* such that

$${}^\circ x^a = x^a(u).$$

The curve C is the projection on V_4 of the curve U of D_{8-1}, locus of all directions tangent to C at its various points. This parametrized representation defining C is described in W_8 by another curve L(u), locus of the *derivate vectors* at u with respect to various points x of C.

In local coordinates for L(u), we thus have:

$$^\circ x^a = \frac{dx^a}{du}\,. \tag{2.6}$$

In these coordinates, we consider a *scalar-valued* function $f(x^a,\, {}^\circ x^a)$ defined on W_8 which is homogeneous and of first degree with respect to ${}^\circ x^a$. On V_4 to the curve C joining the points x^0 onto x^1 there can always be associated the integral expressed in L(u) as:

$$\Phi = \int_{u^0}^{u^1} f\,(x^a,\, {}^\circ dx^a) = \int_{x^0}^{x^1} f\,(x^a,\, dx^a)\,. \tag{2.7}$$

Always in local coordinates, let us now evaluate the variation of Φ with respect to the variable points of C:

$$\delta\Phi = f_{u^1}\,\delta u^1 - f_{u^0}\,\delta u^0 - \int_{u^0}^{u^1} \delta f\, du\,. \tag{2.8}$$

Classically, inspection shows that

$$\int_{u^0}^{u^1} \delta f\, du = \left(\frac{\partial f}{\partial\, {}^\circ x^a}\,\delta x^a\right)_{u=u^0}^{u=u^1} - \int_{u^0}^{u^1} P_a\,\delta x^a du, \tag{2.9}$$

where the P_a are the first members of the Euler equations associated with the function f.

We infer the expression

$$\delta\Phi = \big[\omega(\delta)\big]_{x^1} - \big[\omega(\delta)\big]_{x^0} - \int_{u^0}^{u^1} P_a\,\delta x^a du\,, \tag{2.10}$$

where $\omega(\delta)$ has the form

$$\omega(\delta) = \left(\frac{\partial f}{\partial\, {}^\circ x^a}\right)\delta x^a - \left({}^\circ x^a\,\frac{\partial f}{\partial\, {}^\circ x^a} - f\right)\delta u \tag{2.11}$$

and due to the homogeneity of f, it reduces to

$$\omega(\delta) = \frac{\partial f}{\partial\, {}^\circ x^a}\,\delta x^a. \tag{2.12}$$

The P_a are the components of a covariant vector $\boldsymbol{P}$ which appear as a scalar product in (2.9):

$$\delta\Phi = \big[\omega(\delta)\big]_{x^1} - \big[\omega(\delta)\big]_{x^0} - \int_{u^0}^{u^1} \langle \boldsymbol{P}\,\delta x\rangle\, du\,. \tag{2.13}$$

§2.3.3. Specific variational derivation. Let us apply the above results to the function

$$f = e^K \frac{ds}{du} = e^K \sqrt{g_{ab} \, {}^\circ x^a \, {}^\circ x^b} \,, \tag{2.14}$$

where K is always defined in V_4.

Between two points x^0 and x^1 of V_4 connected by a time-like curve, we set the integral

$$s' = \int_{x^0}^{x^1} e^K ds = \int_{x^0}^{x^1} e^K \sqrt{g_{ab} \, {}^\circ x^a \, {}^\circ x^b} \,. \tag{2.15}$$

Then upon differentiation, we readily infer

$$\left. \begin{aligned} & f \frac{\partial f}{\partial \, {}^\circ x^a} = e^{2K} g_{ab} \, {}^\circ x^b \\ & f \frac{\partial f}{\partial \, x^a} = e^K \left(\partial_a e^K g_{bc} \, {}^\circ x^a \, {}^\circ x^c + \frac{1}{2} \, e^K \partial_a \, g_{bc} \, {}^\circ x^a \, {}^\circ x^c \right) \end{aligned} \right\} . \tag{2.16}$$

We now choose s as the parameter of the curve C, so the vector

$${}^\circ x^a = \frac{dx^a}{ds} = u^a \tag{2.17}$$

is here regarded as the unit vector tangent to C.

Equations (2.16) then reduce to the following expressions

$$\left. \begin{aligned} & \frac{\partial f}{\partial \, {}^\circ x^b} = e^K u^b \\ & \frac{\partial f}{\partial \, x^b} = \frac{1}{2} \, e^K \partial_b \, g_{ad} \, u^a u^d + \partial_b \, e^K \\ & e^K \{_{abd}\} \, u^a u^d + \partial_b \, e^K \end{aligned} \right\} , \tag{2.18}$$

where $\{_{abd}\}$ denotes the Christoffel symbols of the first kind.

In this parametrized formulation, the components P_b of $\boldsymbol{P}$ are written

$$P_b = \frac{d}{ds} \frac{\partial f}{dx^b} - \frac{\partial f}{dx^b} = \frac{d}{ds} (e^K u_b) - e^K \{abd\} \, u^a u^d - \partial_b \, e^K, \tag{2.19}$$

i.e.

$$P_b = e^K (u^a \, \partial_a u_b) - \{abd\} \, u^a u^d - \partial_a e^K (\delta_b^a - u^a u_b) \,, \tag{2.20}$$

hence
$$P_b = e^K (u^a \nabla_a u_b) - (\partial_a K)(\delta_b^a - u^a u_b)\,, \tag{2.21}$$
and (2.13) becomes
$$\delta s' = \big[\omega(\delta)\big]_{x^1} - \big[\omega(\delta)\big]_{x^0} - \int_{s_0}^{s_1} \langle \boldsymbol{P}\,\delta x\rangle\, ds\,, \tag{2.22}$$
where locally:
$$\omega(\delta) = e^K u_a\, dx^a. \tag{2.23}$$

When the curve C varies between two fixed points x^0 and x^1, (2.22) obviously reduces to
$$\delta s' = -\int_{x^0}^{x^1} \langle \boldsymbol{P}\,\delta x\rangle\, ds\,. \tag{2.24}$$

In order to extremalize s', $\boldsymbol{P}$ must be zero, and since $e^K \neq 0$, we have
$$(u^a \nabla_a u_b) - (\partial_a K)(\delta_b^a - u^a u_b) = 0\,. \tag{2.25}$$

Chapter 3. The Geodesic Equations

§3.1. The Riemannian situation. In V_4, the unit vector satisfies
$$g_{ab}\, u^a u^b = g^{ab}\, u_a u_b = 1\,, \tag{3.1}$$
and differentiating, we thus obtain
$$u^b \nabla_a u_b = 0\,. \tag{3.2}$$

Let us now consider the covariant derivative
$$\nabla_a (r\, u^a u_b) = r\,(\partial_b K)\,, \tag{3.3}$$
where r is a scalar.

If we take into account (3.1) and (3.2), the equation (3.3) is equivalent to
$$u^a \nabla_a u_b = (\partial^a K)(g_{ab} - u_a u_b) \tag{3.4}$$
after contracted multiplication by u^b and division by r.

However, (3.3) is just the conservation condition applied to a tensor T_{ab} provided we set
$$r\,(\partial_b K) = \nabla_a (r\,\delta_b^a)\,. \tag{3.5}$$

Explicitly, the tensor T_{ab} can be written:

$$T_{ab} = r\, u_a u_b - p\, g_{ab}\,. \tag{3.6}$$

We note that this equation has the form of the well-known tensor describing a *perfect fluid* with proper density ρ:

$$T_{ab} = (\rho + p)\, u_a u_b - p\, g_{ab} \tag{3.7}$$

with an equation of state $\rho = f(p)$, and with $r = (\rho + p)$, where p is the scalar pressure of the fluid.

Hence,

$$K = \int_{p_0}^{p} \frac{dp}{\rho + p}\,, \tag{3.8}$$

and

$$u^a \nabla_a u_b = (\partial^a K)(g_{ab} - u_a u_b)\,, \tag{3.9}$$

i.e.

$$u^a \nabla_a u_b = (\partial^a K)\, h_{ab}\,, \tag{3.10}$$

where

$$h_{ab} = g_{ab} - u_a u_b \tag{3.11}$$

is the well-known *projection tensor* in the adopted signature.

The 4-vector $\partial^a K$ is regarded as the acceleration of the flow lines given by the pressure gradient orthogonal to those lines.

We can thus draw a first conclusion: equations (2.25) and (3.9) are formally identical. They represent the differential system which the flow lines must satisfy, or in other words, they represent the geodesics of the perfect fluid flow lines.

Theorem: *In a holonomic frame, a perfect fluid follows time-like lines extremalizing the integral*

$$s' = \int_{x^0}^{x^1} e^{2K} ds \tag{3.12}$$

for variations between two fixed points.

Therefore, these flow lines are time-like geodesics conformal to the metric ds^2:

$$(ds^2)' = e^{2K} ds^2 = e^{2K} g_{ab}\, dx^a dx^b \tag{3.13}$$

with the following metric tensor components:

$$(g_{ab})' = e^{2K} g_{ab}\,, \qquad (g^{ab})' = e^{-2K} g^{ab}. \tag{3.14}$$

One can find a similar conclusion in [14] and [15].

§3.2. The EGR geodesics for the EGR massive tensor.

§3.2.1. A conformal like 4-velocity. We now define the collinear vectors w_a with u_a:

$$w_a = e^K u_a \,, \qquad w^a = e^K u^a, \tag{3.15}$$

hence

$$(w_a)' = e^K u_a, \tag{3.16}$$

$$(w^a)' = e^{-K} u^a. \tag{3.17}$$

These are regarded as 4-velocities and are *still* unit vectors in the conformal metric $(ds^2)'$.

As a result, an alternative way of expressing (3.10) can be easily shown to be

$$(w^a)'(\nabla_a)'(w_b)' = 0 \,, \tag{3.18}$$

where $(\nabla_a)'$ is the covariant derivative in $(ds^2)'$ which is built from the conformal connection

$$\left\{{a \atop bc}\right\}' = \left\{{a \atop bc}\right\} + g^{ad}\big(g_{cd} K_b + g_{bd} K_c - g_{bc} K_d\big)\,. \tag{3.19}$$

§3.2.2. Relation between the Weyl and the EGR connections. We recall that the EGR theory is built from two types of curvature forms in a dual basis:

The *rotation curvature 2-form* is:

$$\Omega^a_b = -\frac{1}{2}\, R^{a}_{\cdot bcd}\, \theta^c \wedge \theta^d. \tag{3.20}$$

The *segmental curvature 2-form* is:

$$\Omega = -\frac{1}{2}\, R^{a}_{\cdot acd}\, \theta^c \wedge \theta^d. \tag{3.21}$$

This last form results from the *variation* of the parallely transported vector around a closed path, a feat which necessarily induces $\mathrm{D}_a\, g_{bc} \neq 0$.

Therefore, the global symmetric connection is easily inferred as

$$\Gamma^a_{bc} = \left\{{a \atop bc}\right\} + g^{ad}\big(\mathrm{D}_b\, g_{cd} + \mathrm{D}_c\, g_{bd} - \mathrm{D}_d\, g_{bc}\big) \tag{3.22}$$

and the Weyl connection

$$W^a_{bc} = \left\{{a \atop bc}\right\} + g^{ad}\big(g_{cd} F_d + g_{bd} F_c - g_{bc} F_d\big) \tag{3.23}$$

is simply obtained by setting

$$\mathrm{D}_a\, g_{bc} = g_{bc}\, F_a\,. \tag{3.24}$$

Hence, in this particular case, we can relate the EGR connection to the Weyl counterpart by

$$g_{bc}\, F_a = \mathrm{D}_a\, g_{bc} = \frac{1}{3}\big(\, J_c\, g_{ab} + J_b\, g_{ac} - J_a\, g_{bc}\big). \tag{3.25}$$

§3.2.3. The EGR geodesic equation. In a strict Weyl formulation, the equation (3.4) can be derived up to

$$u^a\, \nabla_a\, u_b = F^a\big(\, g_{ab} - u_a u_b\, \big), \tag{3.26}$$

where the form $dF = F^a dx_a$ is *not integrable*.

The F can always be chosen so that we have the correspondence

$$\left.\begin{aligned} e^F (u_a)_{\mathrm{EGR}} &\longrightarrow (w_a)' = e^K u_a \\ e^{-F} (u^a)_{\mathrm{EGR}} &\longrightarrow (w^a)' = e^{-K} u^a \end{aligned}\right\}. \tag{3.27}$$

Therefore, the EGR geodesic equation for the neutral matter is analogously expressed by

$$(u^a)_{\mathrm{EGR}}\, \mathrm{D}_a (u_b)_{\mathrm{EGR}} = F^a \left[\, g_{ab} - (u_a u_b)_{\mathrm{EGR}}\right]. \tag{3.28}$$

This equation is obeyed by the flow lines of the dynamical mass-gravity field whose energy-momentum tensor is given by

$$(T_{ab})_{\mathrm{EGR}} = \left[\, \rho_{\mathrm{EGR}}\, (u_a u_b)_{\mathrm{EGR}} + (t_{ab})_{\mathrm{field}}\right], \tag{3.29}$$

which is to be compared with the tensor

$$T_{ab} = \big(\rho + p\big)\, u_a u_b - p\, g_{ab}\,. \tag{3.30}$$

Consequences and conclusions. The main goal of our successives studies is to provide a physical justification of the theory that predicts an underlying medium which contains and unifies the particle and antiparticle states, see [16–20].

To sustain this argument, we have asserted that the EGR Theory must exhibit a background persistent field filling the standard physical vacuum.

In addition, when a neutral massive source is present, the persistent field tensor should supersede the matter *gravity pseudo-tensor* necessarily required by the conservation condition which is imposed by the conserved *Riemannian* Einstein tensor.

Therefore, the EGR theory allows for describing the geodesic motion of a dynamical entity that includes the *bare* mass slightly increased by its own surrounding gravity field.

In this paper, we have strictly shown that this is indeed the case, if one places himself in the frame of the generalized variational spaces which are easily related to the EGR manifold.

Using then a holonomic frame of reference, we eventually find that the inferred *Riemannian* perfect fluid tensor matches the model of the EGR energy-momentum tensor of neutral homogeneous matter. In the Riemannian scheme, the role of the persistent field tensor is taken up by the fluid pressure, and the increased bare mass density is here modified by the fluid pressure through an equation of state.

Submitted on March 29, 2012

1. Marquet P. The EGR theory: An extended formulation of general relativity. *The Abraham Zelmanov Journal*, 2009, vol. 2, 150–170.
2. Landau L. D. et Lifshitz E. M. Théorie des Champs. Traduit du Russe par E. Gloukhian, Éditions de la Paix, Moscou, 1964.
3. Marquet P. Behaviour of the EGR persistent vacuum field following the Lichnérowicz matching conditions. *The Abraham Zelmanov Journal*, 2010, vol. 3, 71–80.
4. Lichnérowicz A. Les espaces variationels généralisés. *Ann. Ecole Normale Supérieure*, 3ème série, t. 62, 339–384.
5. Leclerc R. L. Schéma géométrique à tenseur impulsion énergie asymétrique. *Ann. Institut Henri Poincaré, Section A*, 1972, t. XVII, no. 3, 259–289.
6. Schouten J. A. Der Ricci Kalkül. Springer, Berlin, 1924.
7. Marquet P. Geodesics and Finslerian equations in the EGR Theory. *The Abraham Zelmanov Journal*, 2010, vol. 3, 90–101.
8. Leray J. Hyperbolic Differential Equations. The Institute for Advanced Studies — Princeton University Press, Princeton, 1953.
9. Marquet P. An extended theory of GR unifying the matter-anti matter states. *Applied Physics Research*, 2011, vol. 3, 60–75.
10. Marquet P. The EGR field quantization. *The Abraham Zelmanov Journal*, 2011, vol. 4, 162–181.
11. Kramer D., Stephani H., Hertl E., and Mac Callum M. Exact Solutions of Einstein's Field Equations. Cambridge University Press, Cambridge, 1979.
12. Hawking S. W. and Ellis G. F. R. The Large Scale Structure of Space-Time. Cambridge University Press, Cambridge, 1986.

13. Lichnerowicz A. Théories Relativistes de la Gravitation et de l'Electromagnétisme. Masson et Cie, Editeurs, Paris, 1955.

14. Eisenhart L. P. *Trans. Amer. Math. Soc.*, 1924, vol. 26, 205–220.

15. Synge J. L. *Proc. London Math. Soc.*, 1937, vol. 43, 376–416.

16. De Broglie L. Thermodynamique relativiste et mécanique ondulatoire. *Ann. Institut Henri Poincaré*, 1968, vol. IX, no. 2, 89–108.

17. De Broglie L. La dynamique du guidage dans un milieu réfringent et dispersif et la théorie des antiparticules. *Le Journal de Physique*, 1967, t. 28, 481.

18. Marquet P. On the physical nature of the wave function: A new approach through the EGR Theory. *The Abraham Zelmanov Journal*, 2009, vol. 2, 195–207.

19. Marquet P. The matter-antimatter concept revisited. *Progress in Physics*, 2010, vol. 2, 48–54.

20. Vigier J. P. and Bohm D. Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations. *Physical Review*, 1954, vol. 96, no. 1, 208–216.

Vol. 5, 2012 **ISSN 1654-9163**

THE

ABRAHAM ZELMANOV JOURNAL

The journal for General Relativity, gravitation and cosmology

TIDSKRIFTEN

ABRAHAM ZELMANOV

Den tidskrift för allmänna relativitetsteorin, gravitation och kosmologi

The Abraham Zelmanov Journal is a non-commercial, academic journal registered with the *Royal National Library of Sweden*. This journal was typeset using LaTeX typesetting system.

The Abraham Zelmanov Journal är en ickekommersiell, akademisk tidskrift registrerat hos *Kungliga biblioteket*. Denna tidskrift är typsatt med typsättningssystemet LaTeX.