Non-Linear Cosmological Redshift: The Exact Theory According to General Relativity

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Abstract: A new method of calculation is applied to the frequency of a photon according to the travelled distance. It consists in solving the scalar geodesic equation (equation of energy) of the photon, and manifests gravitation, non-holonomity, and deformation of space as the intrinsic geometric factors affecting the photon's frequency. In the space of Schwarzschild's mass-point metric, the well-known gravitational redshift has been obtained. No frequency shift has been found in the rotating space of Gödel's metric, and in the space of Einstein's metric (a homogeneous distribution of ideal liquid and physical vacuum). The other obtained solutions manifest a cosmological effect: its magnitude increases with distance. The parabolic cosmological blueshift has been found in the space of a sphere of incompressible liquid (Schwarzschild's metric), and in the space of de Sitter's metric, which is a sphere filled with physical vacuum whose density is negative (it is a redshift, if the vacuum density is positive). The exponential cosmological redshift has been found in the expanding space of Friedmann's metric (empty or filled with ideal liquid and physical vacuum). This explains the accelerate expanding Universe. The redshift reaches $z = e^{\pi} - 1 = 22.14$ at the event horizon. These results are obtained in a purely geometric way, without the use of the Doppler effect.

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§1. Problem statement. This is the second part of my research, which was started in the publication [1] where I introduced the *cosmological mass-defect* — a new predicted effect revealed according to the General Theory of Relativity. The essence of this effect is that a mass-bearing particle changes its relativistic mass according to the distance travelled by it. The magnitude of this effect can be either positive or negative, depending on the metric of that particular space (the kind of universe) wherein the particle travels.

As was shown, the cosmological mass-defect is obtained after integrating the scalar geodesic equation (equation of energy) of a mass-bearing particle. This equation determines the relativistic energy and mass of the particle at any distance (and moment of time) of its travel. When integrating the equation, the components of the fundamental metric tensor $g_{\alpha\beta}$ are used according to the particular space metric (universe) under consideration. Thus the cosmological mass-defect can be calculated in each particular universe. Following this way, I showed that the cosmological mass-defect is present in most of the main (principal) cosmological models, and provided detailed calculation of its magnitude in each of these cases [1]. In the cosmological models, where this effect is present, the relativistic mass change becomes essential only at distances of the galaxies ("cosmologically large" distances).

This is the cosmological mass-defect in a nutshell. As was pointed out at the end of my previous paper [1], a logical continuation of this research would be solving the scalar geodesic equation of a massless particle (light-like particle, e.g. a photon). As a result, we should expect to obtain the frequency shift of the photon according to the travelled distance in each of the cosmological models.

Naturally, consider the geodesic equations. According to Zelmanov's chronometrically invariant formalism [2–4], any four-dimensional (generally covariant) quantity is presented with its observable projections onto the line of time and the spatial section of an observer*. This is as well true about the generally covariant geodesic equation. The time projection of it is the scalar geodesic equation (equation of energy). The spatial projection is the three-dimensional equation of motion of the particle. As was obtained by Zelmanov [2–4], the projected (chronomet-

*This formalism, known also as the *theory of chronometric invariants*, was introduced in 1944 by Abraham Zelmanov. It is originally given in his primary publications [2–4], while more details of the chronometrically invariant formalism can be found in the books [5, 6]. Chronometric invariance means that the quantity, which possesses this property, is invariant along the observer's three-dimensional spatial section (which can be curved, inhomogeneous, deforming, rotating, etc.).

rically invariant) geodesic equations of a mass-bearing particle, whose relativistic mass is m, are

$$\frac{dm}{d\tau} - \frac{m}{c^2}\,F_i\,\mathrm{v}^i + \frac{m}{c^2}\,D_{ik}\,\mathrm{v}^i\mathrm{v}^k = 0\,, \tag{1.1}$$

$$\frac{d(m\,\mathrm{v}^i)}{d\tau} - m\,F^i + 2\,m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k + m\triangle^i_{nk}\,\mathrm{v}^n\mathrm{v}^k = 0\,, \tag{1.2}$$

while the projected geodesic equations of a massless (light-like) particle (we denote its relativistic frequency as ω) have the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_i\,c^i + \frac{\omega}{c^2}\,D_{ik}\,c^ic^k = 0\,, \tag{1.3}$$

$$\frac{d(\omega c^i)}{d\tau} - \omega F^i + 2\omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k + \omega\triangle^i_{nk}\,c^nc^k = 0\,, \tag{1.4}$$

where the derivation parameter $d\tau = \sqrt{g_{00}}\,dt - \frac{1}{c^2}\,v_i\,dx^i$ is the physically observable (proper) time, which depends on the gravitational potential $\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right)$ and the linear velocity $v_i = -\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ of the three-dimensional rotation of space. The factors affecting the particles are: the gravitational inertial force F_i, the angular velocity A_{ik} of the rotation of space due to its non-holonomity, the deformation D_{ik} of space, and the non-uniformity of space (the Christoffel symbols Δ^i_{jk}). Two factors of these affect the energy of the particle (according to the scalar geodesic equation, which is the equation of energy)

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial\mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \qquad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{1.5}$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \quad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \quad D = h^{ik}D_{ik} = \frac{{}^*\partial\ln\sqrt{h}}{\partial t}\,, \tag{1.6}$$

where $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$, and h_{ik} is the chr.inv.-metric tensor

$$h_{ik} = -\,g_{ik} + \frac{1}{c^2}\,v_iv_k\,, \qquad h^{ik} = -\,g^{ik}, \qquad h^i_k = \delta^i_k\,. \tag{1.7}$$

As is seen, the geodesic equations of mass-bearing and massless particles have the same form. Only the sublight velocity v^i and the relativistic mass m are used for mass-bearing particles instead of the light velocity c^i and the frequency ω of a photon.

It is natural then to suggest that we could solve the scalar geodesic equation of massless particles, which is equation (1.3), in analogy to, as I solved in [1], the scalar geodesic equation of mass-bearing particles (1.1) with the components of the fundamental metric tensor $g_{\alpha\beta}$ according to the respective space metrics (cosmological models).

However, at the end of my previous publication [1], I supposed that this is not a trivial task. My supposition was based on the fact that mass-bearing particles travel in the so-called non-isotropic region of space (space-time), which is the home of the sublight-speed and superluminal trajectories. Massless particles travel in the isotropic space, which is the home of the trajectories of light. The four-dimensional interval is zero everywhere in the isotropic space, while the interval of observable time $d\tau$ and the three-dimensional observable interval $d\sigma^2 = h_{ik}\, dx^i dx^k$ are nonzero and equal to each other

$$\left.\begin{array}{c} ds^2 = c^2 d\tau^2 - d\sigma^2 = 0 \\ c^2 d\tau^2 = d\sigma^2 \neq 0 \end{array}\right\}. \tag{1.8}$$

As a result, the isotropic space (space-time) is strictly non-holonomic: the lines of time meet the three-dimensional coordinate lines at any point therein, and, hence, the isotropic space rotates as a whole at each of its points with the velocity of light (see [7,8] for detail). In terms of the language of algebra, the isotropic space condition, by equalizing the entire formula of $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ to zero, implies an additional relation among the particular components of $g_{\alpha\beta}$ which thus can be transformed into each other in a certain way that does not violate the invariance of the metric as a whole*. This additional condition should be taken into account when considering any problem in the isotropic space. As a result, the scalar equation of isotropic geodesics may have another solution than that obtained after integrating the scalar equation of non-isotropic geodesics. In other words, the frequency shift of photons may have another formulation than the relativistic mass change (mass-defect) of mass-bearing particles travelling in the same space (space-time).

This is why I initially split this study into two parts where, in the first part [1], the scalar geodesic equation of mass-bearing particles is solved, thus introducing the cosmological mass-defect.

However, after studying this problem in detail, I have arrived at another conclusion. Namely — the light-speed rotation, which is at-

*In particular, there should be a replacement among the components g_{00} and g_{0i}. In the case of Minkowski's space, which is the basic space-time of Special Relativity, this replacement means that the isotropic region of it should have the non-diagonal metric where $g_{00} = 0$, $g_{0i} = 1$, and $g_{11} = g_{22} = g_{33} = -1$. Such isotropic metrics were studied in already the 1950's, by Alexei Petrov. See his *Einstein Spaces* [9]. For instance, §25 and the others therein.

tributed to the isotropic space (even in Minkowski's space, which is the basic space-time of Special Relativity) can be registered only by an observer who accompanies the isotropic space and photon. In other words, this is a light-like observer whose home is the isotropic space. A regular (sublight-speed) observer shall observe the isotropic space and all events in it according to the values of the fundamental metric tensor $g_{\alpha\beta}$ which characterize his own (non-isotropic) space — home of "solid objects". This is because, according to the theory of physically observable quantities, an observer should accompany his own reference body and coordinate grids spanned over it. As a result, the isotropic geodesic equations should be solved for a sublight-speed observer in the *same way* as the non-isotropic geodesic equations. In other words, when solving the scalar equation of isotropic geodesics, we should use the components of the fundamental metric tensor which are attributed to the home space (the coordinate grids and clocks) of "solid objects" which is the reference space of a regular observer.

I will give a complete explanation of this thesis later, in one of the chapters of the book on the cosmological mass-defect and the cosmological redshift (now — under preparation).

In the next paragraphs of this paper, after solving the scalar equation of isotropic geodesics in each of the main "cosmological" metrics, we will arrive at the formula of the frequency shift of a photon according to the travelled distance in each of the universes under consideration.

Actually, the method of integration and the solutions will have the same form as those for the cosmological mass-defect obtained for mass-bearing particles in my recent paper [1]. Therefore, to avoid repetition, I will omit some obvious formalities while simply referring to [1] wherein the calculations were explained with all necessary details.

§2. Local redshift in the space of a mass-point (Schwarzschild's mass-point metric). This is the metric of an empty space (in the sense that there is no distributed matter), wherein the field of gravitation and curvature is due to a spherical mass approximated as a mass-point at distances much larger than its radius. The metric, introduced in 1916 by Karl Schwarzschild [10], represented in the spherical three-dimensional coordinates $x^1 = r$, $x^2 = \varphi$, $x^3 = \theta$, has the form

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right), \qquad (2.1)$$

where r is the distance from the mass M, $r_g = \frac{2GM}{c^2}$ is the corresponding gravitational radius of the mass, and G is the world-constant of gravitation.

This metric is free of rotation and deformation. The field of gravitation is the only factor affecting particles in the space. It is determined by g_{00} which is $g_{00} = 1 - \frac{r_g}{r}$ according to the metric (2.1). Differentiating the gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ according to the definition of the gravitational inertial force (1.5), then applying $r \gg r_g$ (the field source is outside the state of gravitational collapse), we obtain the solely nonzero radial component of the force

$$F_1 = -\frac{c^2 r_g}{2r^2} = -\frac{GM}{r^2}\,. \tag{2.2}$$

In such a space, the scalar geodesic equation for a massless (light-like) particle (1.3), e.g. a photon, takes the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_1\,c^1 = 0\,, \tag{2.3}$$

where $c^1 = \frac{dr}{d\tau}$ is the observable velocity of light (massless particle). This equation transforms into $\frac{d\omega}{\omega} = \frac{1}{c^2}F_1\,dr$, which is $d\ln\omega = -\frac{GM}{c^2}\frac{dr}{r^2}$. It has the solution

$$\omega = \omega_0\, e^{\frac{GM}{c^2 r}} \simeq \omega_0\left(1 + \frac{GM}{c^2 r}\right). \tag{2.4}$$

This solution means that a photon emitted by a massive body, which is the field source, gains an additional energy due to the presence of the gravitational field. This phenomenon decreases with distance from the field source according to the formula (2.4). As a result, the photon's energy and frequency should decrease with the travelled distance: the photon's frequency should be redshifted when registered by a observer, who is distantly located from the field source. For instance, let a photon have a wavelength $\lambda_0 = \frac{c}{\omega_0}$ being emitted from the surface of a star, whose mass is M and whose radius is r_*. Then its wavelength registered by an observer, who is located at a distance r from the star, is $\lambda = \frac{c}{\omega}$. We then obtain, according to the formula (2.4), that the observed redshift of the photon has the magnitude

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega}{\omega} = e^{\frac{GM}{c^2 r_*} - \frac{GM}{c^2 r}} - 1 \simeq \frac{GM}{c^2 r_*} - \frac{GM}{c^2 r}\,. \tag{2.5}$$

Note that this is a local phenomenon, not a cosmological effect: its magnitude decreases with distance from the source of the field, very fast, so that it becomes actually zero at "cosmologically large" distances.

This is the gravitational redshift — the well-known effect of the General Theory of Relativity (first registered in the spectra of white dwarfs). I speak of this effect herein, because of the new method of derivation. Classically, it is derived from the conservation of energy of a photon travelling in a stationary gravitational field [11, §88]. The same classical method of derivation was also used by Zelmanov. On the other hand, our method of deduction, based on the integration of the scalar equation of isotropic geodesics, allows to represent this effect as something particular among the other similar effects which can be calculated for any other space metric (gravitational field).

§3. Local redshift in the space of an electrically charged mass-point (Reissner-Nordström metric). This is an extension of the mass-point metric, where the spherical massive island of matter is electrically charged. As a result, the space of the Reissner-Nordström metric is not empty but filled with a spherically symmetric electromagnetic field (distributed matter). This metric was first considered in 1916 by Hans Reissner [12] then, in 1918, by Gunnar Nordström [13]. It is

$$ds^2 = \left(1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right), \tag{3.1}$$

with the same denotations as those of the mass-point metric, while $r_q^2 = \frac{Gq^2}{4\pi\varepsilon_0 c^4}$, where q is the corresponding electric charge, and $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's force constant. This metric is as well free of rotation and deformation. The gravitational inertial force is determined, according to $g_{00} = 1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}$, by both Newton's force and Coulomb's force. Assuming that the source of the field is outside the state of gravitational collapse ($r \gg r_g$), and that the electric field is weak ($r \gg r_q$), we obtain

$$F_1 = -\frac{c^2}{2}\left(\frac{r_g}{r^2} - \frac{2\,r_q^2}{r^3}\right) = -\frac{GM}{r^2} + \frac{Gq^2}{4\pi\varepsilon_0 c^2}\,\frac{1}{r^3}\,. \tag{3.2}$$

The scalar geodesic equation for a massless particle (1.3) takes the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_1\, c^1 = 0\,, \tag{3.3}$$

which transforms into $d\ln\omega = \left(-\frac{GM}{c^2 r^2} + \frac{Gq^2}{4\pi\varepsilon_0 c^4}\frac{1}{r^3}\right) dr$, and solves as

$$\omega = \omega_0\, e^{\frac{GM}{c^2 r} - \frac{1}{2r^2}\frac{Gq^2}{4\pi\varepsilon_0 c^4}} \simeq \omega_0 \left(1 + \frac{GM}{c^2 r} - \frac{1}{2r^2}\,\frac{Gq^2}{4\pi\varepsilon_0 c^4}\right). \tag{3.4}$$

This solution manifests that photons should also gain an additional energy in the field of an electrically charged massive body. But this additional energy is lesser than that gained from the gravitational field (the first term in our formula 3.4), owing to an energy loss due to the presence of the electric field (the negative second term in 3.4).

We observe no such an electrically charged massive body (like planets, stars, or galaxies) whose gravitational field would be weaker than its electromagnetic field. We therefore should conclude that the photon's frequency in the space of the Reissner-Nordström metric should be redshifted when registered by an observer who is located at a distance r from the field source. The redshift, according to our solution (3.4), should be

$$z = \frac{\omega_0 - \omega}{\omega} = e^{\frac{GM}{c^2 r_*} - \frac{1}{2r_*^2}\frac{Gq^2}{4\pi\varepsilon_0 c^4} - \frac{GM}{c^2 r} + \frac{1}{2r^2}\frac{Gq^2}{4\pi\varepsilon_0 c^4}} - 1 \simeq$$
$$\simeq \frac{GM}{c^2 r_*} - \frac{1}{2r_*^2}\frac{Gq^2}{4\pi\varepsilon_0 c^4} - \frac{GM}{c^2 r} + \frac{1}{2r^2}\frac{Gq^2}{4\pi\varepsilon_0 c^4}\,, \qquad (3.5)$$

where r_* is the radius of the field source. This redshift shall be lesser than the purely gravitational redshift (considered in §2).

The magnitude of the redshift decreases with distance from the field source. Therefore, the redshift in the space of the Reissner-Nordström metric is also a local phenomenon, not a cosmological effect.

Herein we have obtained that the frequency shift can be due to not only the field of gravitation, but also due to the electromagnetic field. Such an effect was not considered in the General Theory of Relativity prior to the present study.

Following the same deduction, the frequency shift could also be calculated in two other primary extensions of the mass-point metric. The Kerr metric (introduced in 1963 by Roy P. Kerr [14, 15]) describes the space of a rotating mass-point. The Kerr-Newman metric (introduced in 1965 by Ezra T. Newman [16, 17]) describes the space of a rotating, electrically charged mass-point. However, there is a problem with the calculation. These metrics, determined in the vicinity of the mass-point (field source), do not contain the distribution function of the rotational velocity with distance from the source. Therefore, when integrating the geodesic equation, we should be enforced to introduce these functions on our own behalf (which can be true or false, depending on our understanding of the space rotation). We therefore omit these two cases from consideration.

§4. No frequency shift present in the rotating space with self-closed time-like geodesics (Gödel's metric). This space metric, introduced in 1949 by Kurt Gödel [18], has the form

$$ds^2 = a^2\left[(d\tilde{x}^0)^2 + 2\,e^{\tilde{x}^1}d\tilde{x}^0 d\tilde{x}^2 - (d\tilde{x}^1)^2 + \frac{e^{2\tilde{x}^1}}{2}(d\tilde{x}^2)^2 - (d\tilde{x}^3)^2\right], \quad (4.1)$$

where $a = const > 0$ [cm] is a constant determined as $a^2 = \frac{c^2}{8\pi G\rho} = -\frac{1}{2\lambda}$. Such a space is not empty, but filled with dust of density ρ, and physical vacuum (λ-field). Also, it rotates so that time-like geodesics are self-closed therein.

Gödel's metric was originally given in the form (4.1), through the dimensionless Cartesian coordinates $d\tilde{x}^0 = \frac{1}{a}\,dx^0$, $d\tilde{x}^1 = \frac{1}{a}\,dx^1$, $d\tilde{x}^2 = \frac{1}{a}\,dx^2$, $d\tilde{x}^3 = \frac{1}{a}\,dx^3$, which emphasize the meaning of the world-constant a. We now move to the regular Cartesian coordinates $a\,d\tilde{x}^0 = dx^0 = c\,dt$, $a\,d\tilde{x}^1 = dx^1$, $a\,d\tilde{x}^2 = dx^2$, $a\,d\tilde{x}^3 = dx^3$, which are more suitable for the calculation of the components of the fundamental metric tensor. We obtain

$$ds^2 = c^2dt^2 + 2\,e^{\frac{x^1}{a}}\,c\,dt\,dx^2 - (dx^1)^2 + \frac{e^{\frac{2x^1}{a}}}{2}(dx^2)^2 - (dx^3)^2, \qquad (4.2)$$

where, as is seen,

$$g_{00} = 1\,, \qquad g_{02} = e^{\frac{x^1}{a}}, \qquad g_{01} = g_{03} = 0\,. \qquad (4.3)$$

Therefore the space of Gödel's metric is free of gravitation and deformation, but rotates with a three-dimensional linear velocity $v_i = -\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ whose only nonzero component is

$$v_2 = -\,c\,e^{\frac{x^1}{a}}, \qquad (4.4)$$

which does not depend on time. In this case, the second (inertial) term of the gravitational inertial force F_i (1.5) is zero as well.

All factors which could change the frequency of a photon are absent in the space of Gödel's metric. This means that photons should not gain a frequency shift with the distance travelled therein.

§5. Cosmological blueshift in the space of Schwarzschild's metric of a sphere of incompressible liquid. This metric was introduced in 1916 by Karl Schwarzschild [19] with a limitation imposed on the fundamental metric tensor (he supposed that its three-dimensional components should not possess breaking). This metric in the general form, which is free of the said limitation, was obtained in 2009 by Larissa

Borissova (see formula 3.55 in [20], or (1.1) in [21]). It has the form

$$ds^2 = \frac{1}{4}\left(3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right)^2 c^2dt^2 - \\ - \frac{dr^2}{1-\frac{\varkappa\rho_0 r^2}{3}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (5.1)$$

where $\varkappa = \frac{8\pi G}{c^2}$ is Einstein's gravitational constant, $\rho_0 = \frac{M}{V} = \frac{3M}{4\pi a^3}$ is the density of the liquid, a is the sphere's radius, and r is the radial coordinate within it. The metric is free of rotation and deformation. Only gravitation affects particles due to $g_{00} \neq 1$. Differentiating g_{00} of the metric (5.1), according to the definition of the gravitational inertial force F_i (1.5), we obtain the solely nonzero component of the force

$$F_1 = -\frac{c^2\varkappa\rho_0 r}{3\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\left(3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right)}\,. \qquad (5.2)$$

The scalar geodesic equation for a massless particle (1.3) in this case takes the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}F_1 c^1 = 0\,, \qquad (5.3)$$

which is $d\ln\omega = \frac{1}{c^2}F_1\,dr$. Thus we arrive at the equation

$$d\ln\omega = -\frac{\varkappa\rho_0 r}{3\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}}\,\frac{dr}{\left(3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right)} \qquad (5.4)$$

which transforms into

$$d\ln\omega = -\,d\ln\left(3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right), \qquad (5.5)$$

and solves as

$$\omega = \omega_0\,\frac{3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - 1}{3\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}}\,. \qquad (5.6)$$

Herein $\frac{\varkappa\rho_0 a^2}{3}$ is a world-constant of the space. Generally speaking, its numerical value is permitted to be within the range $0 \leqslant \frac{\varkappa\rho_0 a^2}{3} \leqslant 1$. This is in order to keep $1 - \frac{\varkappa\rho_0 a^2}{3} \geqslant 0$ (the square root from the remain-der should remain a real value). However, in the particular case of the space within a Schwarzschild liquid sphere, we have a possibility to find the exact numerical value of the world-constant $\frac{\varkappa\rho_0 a^2}{3}$.

Determine the radius a of a Schwarzschild liquid sphere as the event horizon of the space. The sphere can be approximated as a mass-point. Its gravitational radius — the distance in the space at which all signals reach the velocity of light, which is the event horizon of the space — is $r_g = \frac{2GM}{c^2}$. Because $\rho_0 = \frac{M}{V} = \frac{3M}{4\pi a^3}$, we obtain $r_g = \frac{\varkappa\rho_0 a^3}{3c^2}$. Assuming $a = r_g$, we obtain $a = \frac{\varkappa\rho_0 a^3}{3c^2}$. Therefore, in the space within a Schwarzschild liquid sphere, the world-constant is

$$\frac{\varkappa\rho_0 a^2}{3} \equiv 1\,. \tag{5.7}$$

With it we obtain, immediately, our solution (5.6) in the final form

$$\omega = \frac{\omega_0}{\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}\,. \tag{5.8}$$

The observed world-density $\rho < 10^{-29}$ gram/cm^3 and Einstein's gravitational constant $\varkappa = \frac{8\pi G}{c^2} = 1.862\times10^{-27}$ cm/gram have very small numerical values. Therefore, at distances r which are very small in comparison to the radius of such a universe ($r \ll a$), the obtained solution (5.8) takes the simplified form

$$\omega \simeq \omega_0 \left(1 + \frac{\varkappa\rho_0 r^2}{6}\right). \tag{5.9}$$

The obtained solution manifests that, in a spherical universe filled with incompressible liquid, a photon should gain energy and frequency with the travelled distance. This is a blueshift effect: the more distant an object we observe in such a universe is, the more blueshifted should be the lines of its spectrum. Hence, this is a cosmological effect. We will therefore further refer to this effect as the *cosmological blueshift*.

According to our formula (5.6), the cosmological blueshift increases with the *square of the distance* from the object. Let the photon have a wavelength $\lambda_0 = \frac{c}{\omega_0}$ being emitted by a source located at a distance r from an observer. Then, keeping in mind that $\lambda = \frac{c}{\omega}$ is the photon's wavelength registered by the observer, we obtain the cosmological blueshift in a spherical universe filled with incompressible liquid. It is

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega}{\omega} = \sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}} - 1\,, \tag{5.10}$$

which at small distances r of the photon's travel ($r \ll a$), according to our formula (5.9), takes the simplified form

$$z \simeq -\frac{\varkappa\rho_0 r^2}{6}\,. \tag{5.11}$$

For a photon emitted by a source, which is located at the event horizon (where $r = a$), the energy and frequency gain is ultimately high in such a universe. According to our solution (5.8), the observed frequency of the photon should be

$$\omega_{\max} = \frac{\omega_0}{\sqrt{1 - \frac{\varkappa\rho_0 a^2}{3}}} = \infty \tag{5.12}$$

while the cosmological blueshift of the photon should take its ultimately high numerical value in such a space, which is

$$z_{\max} = \sqrt{1 - \frac{\varkappa\rho_0 a^2}{3}} - 1 = -1\,. \tag{5.13}$$

In my view, the main criterion for the applicability of a cosmological model to our Universe should be the linear redshift law predicted at small distances $r \ll a$. It is surely registered in most galaxies, which are not so much distant as the event horizon. However, the obtained solution (5.11) manifests the parabolic blueshift. This is a serious reason for us to conclude that Schwarzschild's metric of a sphere of incompressible liquid cannot be applied to our Universe as a whole.

On the other hand, the recent study produced by Borissova [20] manifests that the Schwarzschild model is applicable to the Sun and the planets, by the assumption that these objects can be approximated as spheres of incompressible liquid.

§6. Cosmological blueshift and redshift in the space of a sphere filled with physical vacuum (de Sitter's metric). This metric, introduced in 1917 by Willem de Sitter [22, 23], has the form

$$ds^2 = \left(1 - \frac{\Lambda r^2}{3}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{\Lambda r^2}{3}} - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{6.1}$$

and describes a space filled with a spherically symmetric distribution of physical vacuum (determined by the Λ-term of Einstein's equations). Such a space is free of rotation and deformation, but contains a non-Newtonian gravitational field determined by $g_{00} = 1 - \frac{\Lambda r^2}{3}$ of the metric. Differentiating the g_{00} according to the definition of the gravitational inertial force F_i (1.5), we obtain its solely nonzero component

$$F_1 = \frac{\Lambda c^2}{3}\,\frac{r}{1-\frac{\Lambda r^2}{3}}\,, \tag{6.2}$$

whose magnitude increases with distance. This is a force of repulsion if $\Lambda > 0$ (physical vacuum has a negative density), and is a force of attraction if $\Lambda < 0$ (the vacuum density is positive).*

The scalar geodesic equation for a massless particle (1.3), which in this case has the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_1\,c^1 = 0\,, \tag{6.3}$$

thus transforms into $d\ln\omega = \frac{1}{c^2}F_1\,dr$, which is

$$d\ln\omega = \frac{\Lambda r}{3}\,\frac{dr}{1-\frac{\Lambda r^2}{3}}\,, \tag{6.4}$$

or, in another form,

$$d\ln\omega = -\frac{1}{2}\,d\ln\left(1-\frac{\Lambda r^2}{3}\right). \tag{6.5}$$

This equation solves as

$$\omega = \frac{\omega_0}{\sqrt{1-\frac{\Lambda r^2}{3}}}\,, \tag{6.6}$$

where $\frac{\Lambda r^2}{3}$ should be in the range $0 \leqslant \frac{\Lambda r^2}{3} \leqslant 1$. For yet, observational astronomy provides only information about the upper boundary of the numerical value of the Λ-term, which is as small as $\Lambda \leqslant 10^{-56}\,\text{cm}^{-2}$. Therefore, our obtained solution (6.6) at small distances r takes the simplified form

$$\omega \simeq \omega_0\left(1+\frac{\Lambda r^2}{6}\right). \tag{6.7}$$

The obtained result can lead to two opposite conclusions, depending on the sign of Λ.

Einstein's equations have the form $R_{\alpha\beta} + \frac{1}{2}\,g_{\alpha\beta}\,R = -\varkappa T_{\alpha\beta} + \Lambda\,g_{\alpha\beta}$. Given a space of de Sitter's metric, the Λ-term is connected to the density of physical vacuum ρ, the four-dimensional curvature K, and the three-dimensional observable curvature C as $\rho = -\frac{\Lambda}{\varkappa} = -\frac{3K}{\varkappa} = \frac{C}{2\varkappa}$ (see §5.3 of [5], for details).

*See Chapter 5 of [5], especially §5.3 and §5.5 therein. It is still unclear what sign is really attributed to the Λ-term of Einstein's equations.

Classically, we assume $\Lambda > 0$ so that physical vacuum has negative density and energy as any other potential field. In this case, the non-Newtonian gravitational force is a force of repulsion, the space (space-time) has a positive four-dimensional curvature $K > 0$, while the three-dimensional observable curvature is negative $C < 0$.

Having $\Lambda > 0$, the frequency shift we have obtained in formula (6.6) and in its simplified form (6.7) implies that a photon travelling in a de Sitter universe should gain an additional energy and frequency due to the presence of the non-Newtonian potential field (Λ-field) present in the space. As a result, the photon's frequency should be blueshifted: the more distant an object we observe in a de Sitter universe where $\Lambda > 0$ is, the more blueshifted should be the lines of its spectrum. We will therefore further refer to this effect as the *cosmological blueshift*. The magnitude of the blueshift, according to our solution (6.6), shall be

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega}{\omega} = \sqrt{1 - \frac{\Lambda r^2}{3}} - 1\,, \tag{6.8}$$

where $\lambda_0 = \frac{c}{\omega_0}$ is the wavelength of the photon being emitted by a source, which is located at a distance r from the observer, while the photon's wavelength registered by the observer is $\lambda = \frac{c}{\omega}$. At small distances of the photon's travel, according to (6.7), we have the blueshift

$$z \simeq -\frac{\Lambda r^2}{6}\,. \tag{6.9}$$

At the event horizon — the ultimately large distance $r = a$, which in a de Sitter universe is determined by the obvious condition $\frac{\Lambda r^2}{3} = 1$, — the photon's frequency and blueshift take their ultimately high numerical values. According to our solutions for the photon's frequency (6.6) and its blueshift (6.8), they are

$$\omega_{\max} = \frac{\omega_0}{\sqrt{1 - \frac{\Lambda a^2}{3}}} = \infty\,, \tag{6.10}$$

$$z_{\max} = \sqrt{1 - \frac{\Lambda a^2}{3}} - 1 = -1\,. \tag{6.11}$$

Contrarily, one may assume that the Λ-field is not a potential field but a substance. In this case, it should have positive density and energy, which means that the acting non-Newtonian gravitational force is a force of attraction, the space (space-time) has a negative four-dimensional curvature $K < 0$, its three-dimensional observable curvature is positive

$C > 0$, and also $\Lambda < 0$. In such a de Sitter universe, according to the solutions we have obtained, a photon should lose energy and frequency with the distance travelled by it. This is due to the work spent against the Λ-field. This means that the more distant an object we observe is, the more redshifted should be the lines of its spectrum: the *cosmological redshift* should be registered in a de Sitter universe where $\Lambda < 0$. In this case,

$$\omega = \frac{\omega_0}{\sqrt{1 + \frac{\Lambda r^2}{3}}}\,, \tag{6.12}$$

or, at small distances ($r \ll a$),

$$z \simeq \frac{\Lambda r^2}{6}\,, \tag{6.13}$$

while the ultimate frequency and redshift are

$$\omega_{\max} = \frac{\omega_0}{\sqrt{1 + \frac{\Lambda a^2}{3}}} = \frac{\omega_0}{\sqrt{2}} \simeq 0.71\,\omega_0\,, \tag{6.14}$$

$$z_{\max} = \sqrt{1 + \frac{\Lambda a^2}{3}} - 1 \simeq 0.41\,. \tag{6.15}$$

We however know a number of very distant cosmic objects whose redshift is much higher than $z_{\max} = 0.41$ of the de Sitter model. In the present year, 2011, the highest redshifts registered by the astronomers are $z = 10.3$ (the galaxy UDFj-39546284) and $z = 8.55$ (the galaxy UDFy-38135539). Three dozens of other galaxies and quasars have redshifts higher than $z = 1$. The number of such high redshifted cosmic objects increases with each year of such observations. Also, the parabolic law, which should be expected at small distances of the photon's travel, does not match the linear redshift law registered in most non-extremely distant galaxies. We therefore should unfortunately conclude that our Universe as a whole is not a de Sitter world.

However, we should not be disappointed with the conclusion. As probable, therein are such cosmic objects in our Universe, whose local space can be described by de Sitter's metric. In such a case the de Sitter model (and all our calculations) will be requested. For instance, such ones may be de Sitter black holes: I suggest these are objects filled with physical vacuum, and whose radius meets the respective gravitational radius of such an object. Consider an example of a de Sitter black hole. A de Sitter space, wherein physical vacuum has a positive density $\rho = -\frac{\Lambda}{\varkappa} > 0$ (here $\Lambda < 0$), can be represented as a sphere whose mass is $M = \rho V = \frac{4\pi\rho a^3}{3} = \frac{4\pi\Lambda a^3}{3\varkappa}$. The gravitational radius of a spherical field is $r_g = \frac{2GM}{c^2}$. Thus we obtain $r_g = \frac{\Lambda a^3}{3}$, in our case. A collapsar's radius meets the gravitational radius of its field, i.e. $r_g = a$. As a result, we obtain that $\Lambda = \frac{3}{a^2}$ within a de Sitter black hole. Therefore, considering a de Sitter black hole, we obtain a possibility for calculating the numerical value of the Λ-term within it. For instance, if our Universe (we assume $a = 1.3\times10^{28}$ cm) would be a de Sitter black hole, we would have $\Lambda = \frac{3}{a^2} \simeq 1.8\times10^{-56}\ \text{cm}^{-2}$.

§7. No frequency shift present in the space of a sphere filled with ideal liquid and physical vacuum (Einstein's metric). This metric, introduced in 1917 by Albert Einstein [24], describes a sphere filled with homogeneous and isotropic distribution of ideal (non-viscous) liquid and physical vacuum (Λ-field). It has the form

$$ds^2 = c^2 dt^2 - \frac{dr^2}{1-\Lambda r^2} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{7.1}$$

This metric is free of gravitation ($g_{00} = 1$), rotation ($g_{0i} = 0$), and deformation (the three-dimensional components g_{ik} of the fundamental metric tensor do not depend on time). This means that such a space contains no one factor which could change the frequency of a photon. Hence, the photon's frequency remains unchanged with the distance travelled in the space of Einstein's metric.

§8. Cosmological redshift and blueshift in the deforming spaces of Friedmann's metric. The models, introduced in 1922 by Alexander Friedmann [25, 26], are free of gravitation and rotation, but are deforming, which points to the presence of the functions $g_{ik} = g_{ik}(t)$. In other words, the three-dimensional subspace of the space-time deforms. It may expand, compress, or oscillate. Such a space can be empty, or filled with a homogeneous and isotropic distribution of ideal (non-viscous) liquid in common with physical vacuum (Λ-field), or filled with one of the media. In particular, it can be dust*.

Friedmann's metric has the form

$$ds^2 = c^2 dt^2 - R^2\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right], \tag{8.1}$$

*The energy-momentum tensor of ideal liquid is the same as that of dust except that the latter is marked by the absence of the term containing pressure. In other words, dust is pressureless ideal liquid.

where $R = R(t)$ is the curvature radius of the space, while $\kappa = 0, \pm 1$ is the curvature factor. If $\kappa = -1$, the three-dimensional subspace has the hyperbolic (open) geometry. If $\kappa = 0$, its geometry is flat. If $\kappa = +1$, it has elliptic (closed) geometry. The models with $\kappa = +1$ and $\kappa = -1$ were considered in 1922 and 1924 by Friedmann [25, 26]. The generalized formulation of the metric containing $\kappa = 0, \pm 1$ was first examined in 1925 by Georges Lemaître [27, 28], then in 1929 by Howard Percy Robertson [29], and in 1937 by Arthur Geoffrey Walker [30]. Therefore, Friedmann's metric in its generalized form (8.1) is also known as the Friedmann-Lemaître-Robertson-Walker metric.

Friedmann's metric is expressed here through a "homogeneous" radial coordinate r. It comes across as the regular radial coordinate divided by the curvature radius whose scales change accordingly during expansion or compression of the space. As a result, the homogeneous radial coordinate r does not change its scale with time.

The scalar geodesic equation (1.3) for a massless particle, which travels in a Friedmann universe along the radial coordinate x^1, takes the form

$$\frac{d\omega}{d\tau} + \frac{\omega}{c^2}\, D_{11}\, c^1 c^1 = 0\,, \tag{8.2}$$

where c^1 [sec^{-1}] is the solely nonzero component of the observable "homogeneous" velocity of the massless particle. The square of the velocity is $h_{11}\, c^1 c^1 = c^2$ [cm^2/sec^2]. The components of the chr-inv.-metric tensor h_{ik} (1.7) can be calculated according to Friedmann's metric (8.1). After some algebra, we obtain

$$h_{11} = \frac{R^2}{1 - \kappa r^2}\,, \quad h_{22} = R^2 r^2, \quad h_{33} = R^2 r^2 \sin^2\theta\,, \tag{8.3}$$

$$h = \det \| h_{ik} \| = h_{11}\, h_{22}\, h_{33} = \frac{R^6 r^4 \sin^2\theta}{1 - \kappa r^2}\,, \tag{8.4}$$

$$h^{11} = \frac{1 - \kappa r^2}{R^2}\,, \quad h^{22} = \frac{1}{R^2 r^2}\,, \quad h^{33} = \frac{1}{R^2 r^2 \sin^2\theta}\,. \tag{8.5}$$

In the case of mass-bearing particles, the scalar geodesic equation being in its general form cannot be solved alone. This is because mass-bearing particles can travel at any sub-light velocity, which is unknown. We therefore are enforced to find the velocity by solving the vectorial geodesic equation. This problem was resolved in [1].

Another case — massless (light-like) particles. They travel along isotropic trajectories, which are the trajectories of light. Their velocity $c^i = \frac{dx^i}{d\tau}$ is the observable velocity of light, where $d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i$

is the physically observable time. The observable light velocity c^i depends on the gravitational potential $\mathrm{w} = c^2\left(1-\sqrt{g_{00}}\right)$ and the linear velocity $v_i = -\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ of the three-dimensional rotation of space.

In the case of a Friedmann universe, we have $g_{00}=1$ and $g_{0i}=0$. Hence, $d\tau = dt$ in this case. Thus, because $h_{11}\,c^1c^1 = c^2$, the scalar geodesic equation of a massless particle (8.2) transforms into

$$h_{11}\,\frac{d\omega}{dt} + \omega D_{11} = 0\,, \tag{8.6}$$

thus we obtain $h_{11}\frac{d\omega}{\omega} = -\,D_{11}\,dt$, and, finally, the equation

$$\frac{R^2}{1-\kappa r^2}\;d\ln\omega = -\,D_{11}\,dt\,. \tag{8.7}$$

This equation is non-solvable being considered in the general form as here. To solve this equation, we should simplify it by assuming particular forms of the space deformation (the function $R = R(t)$ of Friedmann's metric) and the curvature factor κ of the space*. Further, after the function $R = R(t)$ is assumed, we will see that κ comes out from the equation. So, we need to assume only $R = R(t)$.

The curvature radius as a function of time, $R = R(t)$, can be found through the tensor of the space deformation D_{ik}, whose trace

$$D = h^{ik}D_{ik} = \frac{^*\partial\ln\sqrt{h}}{\partial t} = \frac{1}{\sqrt{h}}\frac{^*\partial\sqrt{h}}{\partial t} = \frac{1}{V}\frac{^*\partial V}{\partial t} \tag{8.8}$$

is the speed of relative deformation (expansion or compression) of the volume [3, 4]. In an arbitrary metric space, we have

$$D = \frac{1}{V}\frac{^*\partial V}{\partial t} = \gamma\,\frac{1}{a}\frac{^*\partial a}{\partial t} = \gamma\,\frac{u}{a}\,, \tag{8.9}$$

where a is the radius of the volume ($V \sim a^3$), u is the linear velocity of its deformation (positive if the space expands, and negative in the case of compression), and $\gamma = const$ is the shape factor of the space ($\gamma = 3$ in the homogeneous isotropic models [3, 4]).

Two main types of the space deformation, and two respective types of the function $R = R(t)$ were introduced and then examined in [1]. They are as follows.

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*The curvature factor κ is included in the spatial component g_{11} of the fundamental metric tensor of Friedmann's metric (8.1). As a result, and because the space deformation D_{ik} is determined as the time derivative of the three-dimensional components of the observable metric tensor $h_{ik} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k$, the curvature factor κ is included in the formula of the space deformation.

1. A constant-speed deforming (homotachydiastolic) universe* deforms with a constant linear velocity $u = \frac{{}^*\partial a}{\partial t} = const$. Its radius undergoes linear changes with time as $a = a_0 + ut$. Thus

$$D = \gamma \frac{u}{a_0 + ut} \simeq \gamma \frac{u}{a_0}\left(1 - \frac{ut}{a_0}\right), \tag{8.10}$$

where $D = \frac{3\dot{R}}{R}$ as in any Friedmann universe ($\gamma = 3$). Thus we arrive at the equation $\frac{dR}{R} = \frac{u\,dt}{a_0+ut} = \frac{d(a_0+ut)}{a_0+ut}$, which is $d\ln R = d\ln(a_0+ut)$. It solves as $\ln R = \ln|a_0+ut| + \ln B$, i.e. $\frac{R}{B} = a_0 + ut$. The integration constant is found from the condition $R = a_0$ at the initial moment of time $t = t_0 = 0$. It is $B = 1$. Therefore, $R = a_0 + ut$. As a result, we obtain, that in a constant-speed deforming Friedmann universe,

$$R = a_0 + ut\,, \qquad \dot{R} = u\,, \tag{8.11}$$

$$D = \frac{3\dot{R}}{R} = \frac{3u}{a_0 + ut}\,, \tag{8.12}$$

$$D_{11} = \frac{R\dot{R}}{1 - \kappa r^2} = \frac{(a_0 + ut)\,u}{1 - \kappa r^2}\,, \tag{8.13}$$

$$D_1^1 = \frac{\dot{R}}{R} = \frac{u}{a_0 + ut}\,. \tag{8.14}$$

2. In a constant-deformation (homotachydiastolic) universe†, each single volume V (including the total volume of the space), undergoes equal relative changes with time

$$D = \frac{1}{V}\frac{{}^*\partial V}{\partial t} = \gamma\,\frac{u}{a} = const\,, \tag{8.15}$$

while the linear velocity of the deformation increases with time in the case of expansion, and decreases if the space compresses. In other words, this is an accelerate expanding universe or a decelerate compressing universe, respectively.

*I refer to this kind of universe as *homotachydiastolic* (ομοταχυδιαστολικός). Its origin is *homotachydiastoli* — ομοταχυδιαστολή — linear expansion with a constant speed, from όμο which is the first part of όμοιος — the same, ταχύτητα — speed, and διαστολή — linear expansion (compression is the same as negative expansion).

†I refer to this kind of universe as *homotachydioncotic* (ομοταχυδιογκωτικό). This terms originates from *homotachydioncosis* — ομοταχυδιόγκωσης — volume expansion with a constant speed, from όμο which is the first part of όμοιος (omeos) — the same, ταχύτητα — speed, διόγκωση — volume expansion, while compression can be considered as negative expansion.

Generally speaking, a volume element, which is not affected by external factors, expands or compresses so that its volume undergoes equal relative changes with time. We therefore will further consider a *constant-deformation* (*homotachydioncotic*) Friedmann universe.

Because $D=\frac{3\dot{R}}{R}$ in a Friedmann universe, we assume $\frac{\dot{R}}{R}=A=const$ for the constant-deformation (homotachydioncotic) case. We obtain the equation $\frac{1}{R}\,dR=Adt$, which is $d\ln R=Adt$. As a result, in a constant-deformation Friedmann universe whose curvature radius at the present moment of time $t=t_0$ is a_0, we obtain

$$R=a_0\,e^{At}, \qquad \dot{R}=a_0\,A\,e^{At}, \tag{8.16}$$

$$D=\frac{3\dot{R}}{R}=3A=const\,, \tag{8.17}$$

$$D_{11}=\frac{R\dot{R}}{1-\kappa r^2}=\frac{a_0^2\,A\,e^{2At}}{1-\kappa r^2}\,, \tag{8.18}$$

$$D_1^1=\frac{\dot{R}}{R}=A=const\,. \tag{8.19}$$

Thus, substituting $D_{11}=\frac{R\dot{R}}{1-\kappa r^2}=\frac{a_0^2 A e^{2At}}{1-\kappa r^2}$ (8.18) into the scalar geodesic equation (8.7), we obtain the equation in the form

$$d\ln\omega=-A\,dt\,, \tag{8.20}$$

where $A=\frac{\dot{R}}{R}$ is a constant of the space.

As is seen, this equation is independent of the curvature factor κ of the particular Friedmann space under consideration. In other words, by solving this equation we will arrive at a solution which will be common for all three types of the constant-deformation (homotachydiastolic) Friedmann universe which have hyperbolic ($\kappa=-1$), flat ($\kappa=0$), or elliptic ($\kappa=+1$) geometry, respectively.

This equation solves, obviously, as $\ln\omega=-At+\ln B$, where B is an integration constant. So forth, we obtain $\ln\frac{\omega}{B}=-At$, then, trivially, $\omega=B\,e^{-At}$. We calculate the integration constant B from the initial condition $\omega=\omega_0$ at the moment of time $t=t_0=0$. We have $B=\omega_0$. As a result, the final solution of the scalar geodesic equation (8.20) is

$$\omega=\omega_0\,e^{-At}. \tag{8.21}$$

At small distances (and duration) of the photon's travel, the obtained solution takes the simplified form

$$\omega\simeq\omega_0\,(1-At)\,. \tag{8.22}$$

The obtained solution manifests that, in a constant-deformation (homotachydiastolic) Friedmann universe which expands ($A>0$), photons should lose energy and frequency according to the travelled distance. The energy and frequency loss law is exponential (8.21) at large distances of the photon's travel, and is linear (8.22) at small distances.

Accordingly, the photon's frequency should be redshifted. The magnitude of the redshift increases with the travelled distance. This is a *cosmological redshift*, in other words.

Let a photon have an initial wavelength $\lambda_0 = \frac{c}{\omega_0}$ being emitted by a source, with its frequency being registered by an observer who is located at a distance r from the source is $\lambda = \frac{c}{\omega}$. Then we obtain the magnitude of the cosmological redshift in an expanding constant-deformation (homotachydiastolic) Friedmann universe. It is

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega}{\omega} = e^{At} - 1\,, \tag{8.23}$$

which is an *exponential redshift law.* At small distances of the photon travel, it takes the linearized form

$$z \simeq At\,, \tag{8.24}$$

which manifests a *linear redshift law.* Expanding the world-constant $A = \frac{\dot{R}}{R}$ and the duration of the photon's travel $t = \frac{d}{c}$, we have

$$z = e^{\frac{\dot{R}}{R}\frac{d}{c}} - 1\,, \tag{8.25}$$

where $d = ct$ [cm] is the distance to the source emitting the photon. At small distances, we have, respectively, the linear approximation

$$z \simeq \frac{\dot{R}}{R}\,\frac{d}{c}\,. \tag{8.26}$$

In the case where such a universe compresses ($A<0$), this effect changes its sign thus, becoming a *cosmological blueshift.*

Our linearized redshift formula (8.26) is the same as $z = \frac{\dot{R}}{R}\frac{d}{c}$ obtained by Lemaître, the "father" of the theory of an expanding universe who in 1925–1927 discovered the linear redshift law* [28]. He followed, however, another way of deduction which limited him only to the linear formula. He did not arrive at a non-linear generalization of it. Lemaître's beliefs, therefore, remained within the range of the linear redshift law.

*The linear redshift law is now known as *Hubble's law* due to Edwin Hubble's publication of 1929 [32]. See more details about the dramatic history of this discovery in the newest notes [33–35] published in 2011 by the historians of science.

Suppose our world to be an expanding Friedmann universe of the constant-deformation type. Then galaxies should scatter, being carried out with the expanding space. Their spectra should therefore manifest a redshift according to the exponential redshift law (8.25) or, at small distances, according to the linear redshift law (8.26).

The world-constant $A = \frac{\dot{R}}{R}$ can be found on the basis of astronomical observations of the objects whose redshift is within the linear range (the galaxies and quasars which are located not at cosmologically large distances). For instance, consider the brightest quasar 3C 273. Its observed redshift is $z = 0.16$. Such a redshift means that this object is located at a cosmologically small distance (we know distant galaxies and quasars whose redshift is much higher than $z = 1$). Therefore, when calculating the redshift for this object, we use the linearized formula (8.26) of our theory. The observed luminosity distance* to the quasar 3C 273 is $d_L = 749\,\text{Mpc} \simeq 2.3\times10^{27}$ cm. According to our formula (8.26), we obtain that the world-constant $A = \frac{\dot{R}}{R}$ has the numerical value

$$A = \frac{\dot{R}}{R} = z\,\frac{c}{d_L} = 2.1\times10^{-18}\ \text{sec}^{-1}. \tag{8.27}$$

which matches the Hubble constant, which is $H_0 = 72\pm8$ km/sec$\times$Mpc $=$ $= (2.3\pm0.3)\times10^{-18}\ \text{sec}^{-1}$ according to the newest data of the Hubble Space Telescope [31]. The Hubble constant was initially obtained as the coefficient of the observed linear law for scattering galaxies: this law says that galaxies and quasars scatter with the radial velocity $u = H_0 d$ increasing with the distance d to the object as 72 km/sec per each megaparsec.

The ultimately high redshift $z_{\max}$, which could be registered in our Universe, is calculated by substituting the ultimately large distance into the redshift law. If following Lemaître's theory [28], $z_{\max}$ should follow from the linear redshift law $z = \frac{\dot{R}}{R}\frac{d}{c} = A\frac{d}{c}$. Because $A = \frac{\dot{R}}{R}$ is the world-constant of the Friedmann space, the ultimately large curvature radius $R_{\max}$ is determined by the ultimately high velocity of the space expansion which is the velocity of light $\dot{R}_{\max} = c$. Hence, $R_{\max} = \frac{c}{A}$. The ultimately large distance $d_{\max}$ (the event horizon) is determined by the astronomers from the linear law for scattering galaxies $u = H_0 d$. This linear law is known, however, due to the observation of non-extremely distant objects. They thus interpolate the empirical linear law $u = H_0 d$

*In observational astronomy, the luminosity distance d_L to a cosmic object is determined through the absolute stellar magnitude $\mathfrak{M}$ of the object, and its apparent stellar magnitude $\mathfrak{m}$ according to the formula $\mathfrak{M} = \mathfrak{m} - 5\,(\lg d_L - 1)$, where d_L is measured in parsecs. 1 parsec $= 3.0857\times10^{18}$ cm $\simeq 3.1\times10^{18}$ cm.

upto the event horizon. Since the scattering velocity u should reach the velocity of light ($u=c$) at the event horizon ($d=d_{\max}$), they then obtain $d_{\max}=\frac{c}{H_0}=(1.3\pm0.2)\times10^{28}$ cm. Finally, they identify the linear coefficient H_0 of the empirical law for scattering galaxies as the world-constant $A=\frac{\dot{R}}{R}$, which follows from the space geometry. Thus they may obtain $d_{\max}=R_{\max}$ and, from the linear redshift law, the ultimately high redshift $z_{\max}=H_0\frac{d_{\max}}{c}=1$. How, then, to explain the very distant objects, whose redshift is much higher than $z=1$?

On the other hand, it is obvious that the ultimately high redshift $z_{\max}$, ensuing from the space (space-time) geometry, should be a result of relativistic physics. In other words, $z=z_{\max}$ should follow not from a straight line $z=\frac{\dot{R}}{R}\frac{d}{c}=H_0\frac{d}{c}=\frac{u}{c}$, which digs in the vertical "wall" $u=c$, but from a non-linear relativistic function.

In this case, the Hubble constant H_0 remains a linear coefficient in only the pseudo-linear beginning of the real redshift law arc, wherein the velocities of scattering u are small in comparison with the velocity of light. At velocities of scattering close to the velocity of light (close to the event horizon), the Hubble constant H_0 loses the meaning of the linear coefficient and the world-constant A due to the increasing non-linearity of the real redshift law.

Such a non-linear formula has been found in the framework of our theory presented here. This is the exponential redshift law (8.25), which then gives the Lemaître linear redshift law (8.26) as an approximation at small distances.

We now use the exponential redshift law (8.25). We calculate the ultimately high redshift $z_{\max}$, which could be conceivable in an expanding Friedmann space of the constant-deformation type. The event horizon $d=d_{\max}$ is determined by the world-constant $A=\frac{\dot{R}}{R}$ of such a space. Thus, the ultimately large curvature radius is $R_{\max}=\frac{c}{A}$, while the distance corresponding to $R_{\max}$ on the hypersurface is $d_{\max}=\pi R_{\max}=\frac{\pi c}{A}$. Suppose now that a photon has arrived from a source, which is located at the event horizon. According to the obtained exponential solution (8.21), the photon's frequency at the arrival should be

$$\omega_{\max}=\omega_0\, e^{-\frac{\dot{R}}{R}\frac{d_{\max}}{c}}=\omega_0\, e^{-\pi}\simeq 0.043\,\omega_0\,, \tag{8.28}$$

while the exponential redshift law (8.25) gives the photon's redshift

$$z_{\max}=e^{\frac{\dot{R}}{R}\frac{d_{\max}}{c}}-1=e^{\pi}-1=22.14\,, \tag{8.29}$$

which is the ultimately high redshift in such a universe.

So, the redshift law for scattering galaxies, including its non-linear increase at "cosmologically large" distances, has been explained in the expanding constant-deformation (homotachydioncotic) space, which is an accelerate expanding Friedmann universe.

The deduced exponential law points out the ultimately high redshift $z_{\max} = 22.14$ for the objects located at the event horizon. The highest redshifted objects, registered by the astronomers, are now the galaxies UDFj-39546284 ($z = 10.3$) and UDFy-38135539 ($z = 8.55$). According to the theory, they are still distantly located from the "world end". We therefore shall expect, with years of further astronomical observation, more "high redshifted surprises" which will approach the upper limit $z_{\max} = 22.14$ predicted by our theory.

§9. A note on the cosmological mass-defect in a Friedmann universe. In §9 of the previous publication [1], I suggested solving the scalar geodesic equation of mass-bearing particles in a Friedmann universe. This equation being in its general form

$$\frac{dm}{d\tau} + \frac{m}{c^2}\, D_{11}\, \mathrm{v}^1 \mathrm{v}^1 = 0\,, \tag{9.1}$$

is non-resolvable. This is because mass-bearing particles can travel at any sub-light velocity, which is therefore an unknown term of the equation*. I then looked for the velocity by solving the vectorial geodesic equation of mass-bearing particles. As a result, I arrived at a non-resolvable integral equation. Even qualitative analysis of the integral did not give a definite conclusion.

I now understand my mistake in that way of deduction. I targeted that problem in its general form. However, now I see that the problem can easily be removed in a constant-deformation Friedmann universe, where massive bodies (mass-bearing particles) travel not arbitrarily, but are only carried out with the expanding (or compressing) Friedmann space itself. In this particular case, the linear velocity of a mass-bearing particle is the same as the speed $\dot{R}$ at which the curvature radius R of the space changes with time, $\mathrm{v} = \dot{R}$. In other words, because $\mathrm{v}^2 = h_{ik}\, \mathrm{v}^i \mathrm{v}^k$, we have $h_{ik}\, \mathrm{v}^i \mathrm{v}^k = \dot{R}^2$. In this particular case (and with $d\tau = dt$ according to Friedmann's metric), the scalar geodesic equation of mass-bearing particles (9.1) takes the form

$$h_{11}\, \frac{dm}{dt} + \frac{m}{c^2}\, D_{11}\, \dot{R}^2 = 0\,, \tag{9.2}$$

*Massless particles travel at the velocity of light $\mathrm{v}^i = c^i$, so we have not this problem when considering the geodesic equations of massless particles.

which is $h_{11}\frac{dm}{m}=-\frac{\dot{R}^2}{c^2}D_{11}\,dt$, and, finally,

$$\frac{R^2}{1-\kappa r^2}\,d\ln m=-\frac{\dot{R}^2}{c^2}\,D_{11}\,dt\,. \tag{9.3}$$

Then, substituting $R=a_0\,e^{At}$ and $\dot{R}=a_0 A\,e^{At}$ (8.16), and $D_{11}=$ $=\frac{R\dot{R}}{1-\kappa r^2}=\frac{a_0^2 A e^{2At}}{1-\kappa r^2}$ (8.18) as for a constant-deformation space, we obtain the scalar geodesic equation in the form

$$d\ln m=-\frac{a_0^2 A^3 e^{2At}}{c^2}\,dt\,, \tag{9.4}$$

or $d\ln m=-\frac{a_0^2 A^2}{2c^2}\,de^{2At}$, where $A=\frac{\dot{R}}{R}$ is a constant of the space.

Note that the curvature factor κ comes out from the obtained equation. Therefore, the further solution of the equation will be common for all three types of the constant-deformation (homotachydiastolic) Friedmann universe: the hyperbolic ($\kappa=-1$), flat ($\kappa=0$), and elliptic ($\kappa=+1$) space.

This equation solves, obviously, as $\ln m=-\frac{a_0^2 A^2}{2c^2}\,e^{2At}+\ln B$, where the integration constant B can be found from the condition $m=m_0$ at the initial moment of time $t=t_0=0$. Thus, after some trivial algebra, we obtain the final solution of the scalar geodesic equation (9.4). It is the double-exponent

$$m=m_0\,e^{-\frac{a_0^2 A^2}{2c^2}\left(e^{2At}-1\right)}, \tag{9.5}$$

where t is the duration of the expansion (if $A>0$) or compression ($A<0$) of the Friedmann universe. At small distances (and durations of time), this solution takes the linearized form

$$m\simeq m_0\left(1-\frac{a_0^2 A^3\,t}{c^2}\right). \tag{9.6}$$

The obtained exact solution (9.5) and its linearized form (9.6) manifest the *cosmological mass-defect* in a constant-deformation (homotachydiastolic) Friedmann universe: the more distant an object we observe in an expanding Friedmann universe is, the less should be its observed mass m to its real mass m_0. Contrarily, the more distant an object we observe in a compressing Friedmann universe is, the heavier should be this object according to the observation.

Our Universe seems to be expanding. This is due to the cosmological redshift registered in the distant galaxies and quasars. Therefore, according to the cosmological mass-defect deduced here, we should expect distantly located cosmic objects to be much heavier than we estimate on the basis of astronomical observations. The magnitude of the expected mass-defect should be, according to the obtained solutions, in the order of the redshift of the objects.

The cosmological mass-defect complies with the respective solution obtained for the frequency of a photon. Both effects are deduced in the same way, by solving the scalar geodesic equation for mass-bearing and massless particles, respectively. One effect cannot be in the absence of the other, because the geodesic equations have the same form. This is a basis of the space (space-time) geometry, in other words. Therefore, once the astronomers register the linear redshift law and its non-linearity in the very distant galaxies and quasars, they should also find the corresponding cosmological mass-defect according to the solutions outlined here. Once the cosmological mass-defect is discovered, we will be able to say, surely, that our Universe as a whole is an expanding Friedmann universe of the constant-deformation (homotachydiastolic) type.

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P.S. A thesis of this presentation has been posted on the desk of the *April Meeting 2012 of the APS*, planned for March 31 – April 03, 2012, in Atlanta, Georgia. More detailed explanation of the cosmological redshift and the cosmological mass defect, surveyed briefly in my recent papers, will be considered in my forthcoming book (under preparation).

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1. Rabounski D. Cosmological mass-defect — a new effect of General Relativity. *The Abraham Zelmanov Journal*, 2011, vol. 4, 137–161.
2. Zelmanov A. L. Chronometric invariants and accompanying frames of reference in the General Theory of Relativity. *Soviet Physics Doklady*, 1956, vol. 1, 227–230 (translated from *Doklady Academii Nauk USSR*, 1956, vol. 107, no. 6, 815–818).
3. Zelmanov A. L. Chronometric Invariants: On Deformations and the Curvature of Accompanying Space. Translated from the preprint of 1944, American Research Press, Rehoboth (NM), 2006.
4. Zelmanov A. L. On the relativistic theory of an anisotropic inhomogeneous universe. *The Abraham Zelmanov Journal*, 2008, vol. 1, 33–63 (originally presented at the *6th Soviet Meeting on Cosmogony*, Moscow, 1959).
5. Borissova L. and Rabounski D. Fields, Vacuum, and the Mirror Universe. Svenska fysikarkivet, Stockholm, 2009.
6. Rabounski D. and Borissova L. Particles Here and Beyond the Mirror. Svenska fysikarkivet, Stockholm, 2008.
7. Rabounski D. Hubble redshift due to the global non-holonomity of space. *The Abraham Zelmanov Journal*, 2009, vol. 2, 11–28.

8. Rabounski D. On the speed of rotation of the isotropic space: insight into the redshift problem. *The Abraham Zelmanov Journal*, 2009, vol. 2, 208–223.
9. Petrov A. Z. Einstein Spaces. Pergamon Press, Oxford, 1969 (translated by R. F. Kelleher, edited by J. Woodrow).
10. Schwarzschild K. Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 1916, 189–196 (published in English as: Schwarzschild K. On the gravitational field of a point mass according to Einstein's theory. *The Abraham Zelmanov Journal*, 2008, vol. 1, 10–19).
11. Landau L. D. and Lifshitz E. M. The Classical Theory of Fields. 4th expanded edition, translated by M. Hammermesh, Butterworth-Heinemann, 1980.
12. Reissner H. Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie. *Annalen der Physik*, 1916, Band 50 (355), no. 9, 106–120.
13. Nordström G. On the energy of the gravitational field in Einstein's theory. *Koninklijke Nederlandsche Akademie van Wetenschappen Proceedings*, 1918, vol. XX, no. 9–10, 1238–1245 (submitted on January 26, 1918).
14. Kerr R. P. Gravitational field of a spinning mass as an example of algebraically special metrics. *Physical Review Letters*, 1963, vol. 11, no. 5, 237–238.
15. Boyer R. H. and Lindquist R. W. Maximal analytic extension of the Kerr metric. *Journal of Mathematical Physics*, 1967, vol. 8, no. 2, 265–281.
16. Newman E. T. and Janis A. I. Note on the Kerr spinning-particle metric. *Journal of Mathematical Physics*, 1965, vol. 6, no. 6, 915–917.
17. Newman E. T., Couch E., Chinnapared K., Exton A., Prakash A., Torrence R. Metric of a rotating, charged mass. *Journal of Mathematical Physics*, 1965, vol. 6, no. 6, 918–919.
18. Gödel K. An example of a new type of cosmological solutions of Einstein's field equations of gravitation. *Reviews of Modern Physics*, 1949, vol. 21, no. 3, 447–450.
19. Schwarzschild K. Über das Gravitationsfeld einer Kugel aus incompressiebler Flüssigkeit nach der Einsteinschen Theorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 1916, 424–435 (published in English as: Schwarzschild K. On the gravitational field of a sphere of incompressible liquid, according to Einstein's theory. *The Abraham Zelmanov Journal*, 2008, vol. 1, 20–32).
20. Borissova L. The gravitational field of a condensed matter model of the Sun: The space breaking meets the Asteroid strip. *The Abraham Zelmanov Journal*, 2009, vol. 2, 224–260.
21. Borissova L. De Sitter bubble as a model of the observable Universe. *The Abraham Zelmanov Journal*, 2010, vol. 3, 3–24.
22. De Sitter W. On the curvature of space. *Koninklijke Nederlandsche Akademie van Wetenschappen Proceedings*, 1918, vol. XX, no. 2, 229–243 (submitted on June 30, 1917, published in March, 1918).
23. De Sitter W. Einstein's theory of gravitation and its astronomical consequences. Third paper. *Monthly Notices of the Royal Astronomical Society*, 1917, vol. 78, 3–28 (submitted in July, 1917).
24. Einstein A. Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. *Sitzungsberichte der Königlich preussischen Akademie der Wissenschaften zu Berlin*, 1917, 142–152 (eingegangen am 8 February 1917).

25. Friedmann A. Über die Krümmung des Raumes. *Zeitschrift für Physik*, 1922, Band 10, No. 1, 377–386 (published in English as: Friedman A. On the curvature of space. *General Relativity and Gravitation*, 1999, vol. 31, no. 12, 1991–2000).

26. Friedmann A. Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes. *Zeitschrift für Physik*, 1924, Band 21, No. 1, 326–332 (published in English as: Friedmann A. On the possibility of a world with constant negative curvature of space. *General Relativity and Gravitation*, 1999, vol. 31, no. 12, 2001–2008).

27. Lemaître G. Note on de Sitter's universe. *Journal of Mathematical Physics*, 1925, vol. 4, 188–192.

28. Lemaître G. Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Societe Scientifique de Bruxelles*, ser. A, 1927, tome 47, 49–59 (published in English, in a substantially shortened form — we therefore strictly recommend to go with the originally publication in French, — as: Lemaître G. Expansion of the universe, a homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ. *Monthly Notices of the Royal Astronomical Society*, 1931, vol. 91, 483–490).

29. Robertson H. P. On the foundations of relativistic cosmology. *Proceedings of the National Academy of Sciences of the USA*, 1929, vol. 15, no. 11, 822–829.

30. Walker A. G. On Milne's theory of world-structure. *Proceedings of the London Mathematical Society*, 1937, vol. 42, no. 1, 90–127.

31. Freedman W. L., Madore B. F., Gibson B. K., Ferrarese L., Kelson D. D., Sakai S., Mould J. R., Kennicutt R. C. Jr., Ford H. C., Graham J. A., Huchra J. P., Hughes S. M. G., Illingworth G. D., Macri L. M., Stetson P. B. Final results from the Hubble Space Telescope Key Project to measure the Hubble constant. *Astrophys. Journal*, 2001, vol. 553, issue 1, 47–72.

32. Hubble E. A relation between distance and radial velocity among extra-galactic nebulae. *Proceedings of the National Academy of Sciences of the USA*, 1929, vol. 15, 168–173.

33. van den Bergh S. The curious case of Lemaître's equation no. 24. Cornell University arXiv: 1106.1195 (2011).

34. Block D. L. A Hubble eclipse: Lemaître and censorship. Cornell University arXiv: 1106.3928 (2011).

35. Reich E. S. Edwin Hubble in translation trouble. Amateur historians say famed astronomer may have censored a foreign rival. *Nature News*, 27 June 2011.

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