The EGR Field Quantization

Patrick Marquet*

Abstract: In this paper, we show that the EGR curvature tensor can be quantized according to the procedure set forth by André Lichnérowicz which relies on the definition of tensor propagators. This quantization is here successfully applied to a space-time with constant curvature defined in the framework of the EGR Theory. Having then extended the initial Einstein space, it implies ipso facto the existence of a generalized cosmological constant which thereby finds here a full physical justification.

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*Postal address: 7, rue du 11 nov, 94350 Villiers/Marne, Paris, France. E-mail: patrick.marquet6@wanadoo.fr. Tel: (33) 1-49-30-33-42.

Introduction. General Relativity and quantum field theories are still the greatest achievements of present-time physics. Although the second part of our last century has seen some significant progresses, quantization rules in a curved space (space-time) background remain a never-ending unfinished story.

To date, it seems that André Lichnérowicz remains the pioneer who first succeeded in applying the regular commutation rules to the gravitational field in a constant-curvature space. Following the standard procedure applied to the electromagnetic field in the Minkowski space, Lichnérowicz formally showed that the *varied* Riemannian curvature tensor can also be quantized in the particular case of an Einstein space with constant curvature. This essential work was published in three communications to the French Academy of Sciences [1–3]. Those were lectured at the Collège de France in Paris, during the year 1958–1959.

The quantization rules, which were formulated by Lichnérowicz, state that:

a) The gravitational field is entirely described by the Riemann curvature tensor;

b) By strict analogy with the electromagnetic field, the *varied* curvature tensor can be adequately quantized in the Minkowski space and by continuity in constant-curvature space.

In a curved space-time, the adopted procedure requires the use of *tensor propagators* associated with second-order differential operators (Lichnérowicz [4]). Such propagators are based on the concept of *displacement bi-tensors* and are analogous to the Green functions introduced by Bryce de Witt et al., during the same period [5]. In this paper, we will only restrict our study to the related general definitions and we invite the reader to the referred bibliography for deeper mathematical analysis.

In the EGR framework (Marquet [6]), the EGR field equations always retain a true background persistent field tensor $(t^{ab})_{\text{field}}$ that *super-*

sedes the ill-defined energy-momentum pseudo-tensor of a mass gravitational field t^{ab} required to satisfy the conservation law within the Riemannian physics. In the absence of substance (source-free field equations), the persistent field can be formally merged into a *generalized cosmological term*, thus allowing the definition of a *EGR Einstein space*.

With this preparation, we are able to extend here the procedure already developed in the Riemannian framework. The quantization rules (commutators) are next applied to a *massless varied* EGR field tensor defined within the EGR Einstein space, and by doing so, the inferred EGR second-order curvature tensor becomes symmetric. As a result, all existing differential operations still hold, and a similar commutator for the varied EGR 4th-rank tensor can be derived in the EGR constant-curvature space.

Chapter 1. Some Topics within EGR Theory

§1.1. The EGR manifold

§1.1.1. The EGR field equations

We briefly recall here our previous results needed for the clarity of this paper.

On the EGR manifold M, are defined the components of the EGR curvature tensor

$$(R^{a\cdots}_{\cdot bcf})_{\text{EGR}} = \partial_c \Gamma^a_{bf} - \partial_f \Gamma^a_{bc} + \Gamma^a_{dc} \Gamma^d_{bf} - \Gamma^a_{df} \Gamma^d_{bc}$$

with the EGR semi-affine connection

$$\Gamma^d_{ab} = \{^{\,d}_{ab}\} + (\Gamma^d_{ab})_J \,, \tag{1.1}$$

where $\{^{\,d}_{ab}\}$ are the regular Christoffel symbols and

$$(\Gamma^d_{ab})_J = \frac{1}{6}\left(\delta^d_a J_b + \delta^d_b J_a - 3\, g_{ab} J^d\right).$$

As to the physical interpretation of the vector J^a, one can refer to the explanation given in the earlier publication [7].

The EGR covariant derivative denoted hereinafter by D or $'$, applies to the metric as

$$\mathrm{D}_a\, g_{bc} = \partial_a\, g_{bc} - \Gamma^f_{ba}\, g_{fc} - \Gamma^f_{ca}\, g_{bf} = \frac{1}{3}\left(J_c\, g_{ab} + J_b\, g_{ac} - J_a\, g_{bc}\right). \tag{1.2}$$

The second-order curvature tensor

$$(R_{bc})_{\text{EGR}} = \partial_a \Gamma^a_{bc} - \partial_c \Gamma^a_{ba} + \Gamma^d_{bc} \Gamma^a_{da} - \Gamma^d_{ba} \Gamma^a_{dc} \tag{1.3}$$

reveals its *non-symmetric* property*

$$(R_{ab})_{\text{EGR}} = R_{ab} - \frac{1}{2}\left(g_{ab}\,\nabla_e J^e + \frac{1}{3}\,J_a J_b\right) + \frac{1}{6}\left(\partial_a J_b - \partial_b J_a\right) \quad (1.4)$$

and leads to the *EGR Einstein tensor*

$$(G_{ab})_{\text{EGR}} = (R_{ab})_{\text{EGR}} - \frac{1}{2}\left(g_{ab}\,R_{\text{EGR}} - \frac{2}{3}\,J_{ab}\right) \quad (1.5)$$

with the *EGR curvature scalar*

$$R_{\text{EGR}} = R - \frac{1}{3}\left(\nabla_e J^e + \frac{1}{2}\,J^2\right). \quad (1.6)$$

The EGR theory allows for a *vacuum persistent field* to pre-exist, which appears in the *source-free EGR field equations*

$$(G_{ab})_{\text{EGR}} = (R_{(ab)})_{\text{EGR}} - \frac{1}{2}\left(g_{ab}\,R_{\text{EGR}} - \frac{2}{3}\,J_{ab}\right) = \varkappa\,(t_{ab})_{\text{EGR}}\,, \quad (1.7)$$

where $\varkappa = \frac{8\pi\mathfrak{G}}{c^4}$ is the Einstein constant and $\mathfrak{G}$ is the Newton constant.

When a massive (anti-symmetric) tensor $T_{ab}(\rho)$ is present on the right-hand side, we have the EGR field equations

$$(G_{ab})_{\text{EGR}} = \varkappa\left[\,T_{ab}(\rho) + (t_{ab})_{\text{EGR}}\,\right]. \quad (1.8)$$

In the EGR theory, the mass density ρ is now increased by its own gravity field precisely due to the continuity of the persistent field $(t_{ab})_{\text{EGR}}$ (Marquet [8]). The EGR formulation is therefore a theory which is capable of describing a dynamical entity (massive particle together with its gravity field), that follows a geodesic distinct from the Riemannian geodesic. Accordingly, the isotropic vectors on M are slightly modified, as we will see below.

§1.1.2. The EGR line element

On the manifold M, the *isotropic conoids* as they are defined in the Riemannian picture, do not exactly coincide with the EGR representation, because the EGR line-element slightly deviates from the standard Einstein geodesic invariant.

*We denote covariant derivative on the Riemannian manifold V_4 by ∇_a or ; while keeping denotation D_a for covariant derivative on M.

Indeed, consider the vector $\boldsymbol{l}$ whose square is given by

$$\boldsymbol{l}^2 = g_{ab}\, A^a A^b. \tag{1.9}$$

Along an infinitesimal closed path, this vector will now *vary* when parallel transported according to

$$d\boldsymbol{l}^2 = d(g_{ab})\, A^a A^b + g_{ab}(dA^a)^{\parallel} A^b + g_{ab}\, A^a (dA^b)^{\parallel} = \\ = \left(dg_{ab} - \Gamma^c_{ad}\, dx^d g_{cb} - \Gamma^c_{bd}\, dx^d g_{ac}\right) A^a A^b \tag{1.10}$$

since

$$(dA^a)^{\parallel} = -\Gamma^a_{id}\, A^i dx^d \tag{1.11}$$

with the EGR semi-affine connection defined above (1.1).

From the general definition of the covariant derivative of the metric tensor (1.2)

$$\mathrm{D}_d\, g_{ab} = \partial_d\, g_{ab} - \Gamma^c_{ad}\, g_{cb} - \Gamma^c_{bd}\, g_{ac} \tag{1.12}$$

we write the differential as

$$\mathrm{D}\, g_{ab} = dg_{ab} - (\Gamma^c_{ad}\, g_{cb} - \Gamma^c_{bd}\, g_{ac})\, dx^d \tag{1.13}$$

so, we have

$$d\boldsymbol{l}^2 = (\mathrm{D}\, g_{ab})\, A^a A^b, \tag{1.14}$$

$$d\boldsymbol{l}^2 = (\mathrm{D}_d\, g_{ab})\, A^a A^b dx^d. \tag{1.15}$$

The EGR line-element includes a small correction due to the Riemannian invariant ds^2

$$(ds^2)_{\mathrm{EGR}} = ds^2 + d(ds^2)\,. \tag{1.16}$$

Therefore, we have

$$d(ds^2) = d\left(g_{ab}\, dx^a dx^b\right). \tag{1.17}$$

Taking then into account (1.13), we find

$$d(ds^2) = \left(\partial_d\, g_{ab} - \Gamma_{ad,b} - \Gamma_{bd,a}\right) dx^a dx^b dx^d, \tag{1.18}$$

or

$$d(ds^2) = \left(\mathrm{D}_d\, g_{ab}\right) dx^a dx^b dx^d \tag{1.19}$$

having noted that

$$\Gamma_{ab,i} = g_{id}\, \Gamma^d_{ab}\,. \tag{1.20}$$

Eventually

$$d(ds^2) = (\mathrm{D}\, g_{ab})\, dx^a dx^b \tag{1.21}$$

with $\mathrm{D}\, g_{ab} = \frac{1}{3}\,(J_c\, g_{ab} + J_b\, g_{ac} - J_a\, g_{bc})\, dx^c$.

Hence, the EGR line-element is simply expressed by

$$(ds^2)_{\text{EGR}} = (\,g_{ab} + \mathrm{D}\, g_{ab})\, dx^a dx^b, \tag{1.22}$$

which naturally reduces to the Riemannian (invariant) interval ds^2 when the covariant derivative of the metric tensor g_{ab} vanishes (i.e. in the case where $J_a = 0$).

The form of the second term (correction) is legitimate since it must exhibit the metric covariant variation that corresponds to the parallel transported variable vector, in contrast to Riemannian geometry. Thus, the EGR conoids $\mathrm{C}^{\pm}_{\text{EGR}}$, which will be used hereinafter, do not exactly coincide with the Riemannian conoids $\mathrm{C}^{\pm}$.

§1.2. The constant-curvature space in the EGR Theory

§1.2.1. Definitions

In the Riemannian framework, it is well known that the four-dimensional space-time metric with constant curvature is

$$R_{abed} = K\left(g_{ae}\, g_{bd} - g_{ad}\, g_{be}\right) \tag{1.23}$$

with

$$K = \frac{R}{12}\,, \tag{1.24}$$

where R is the *constant* curvature scalar. If $K = \frac{\lambda}{3}$, where λ is the cosmological constant, the constant-curvature Riemannian manifold V_4 is the so-called *Einstein space* (see, for instance, the explanation given by L. Borissova and D. Rabounski [9], formulae 5.33–5.34). In this case, one writes

$$G_{ab} = R_{ab} = \lambda\, g_{ab}\,. \tag{1.25}$$

§1.2.2. The Einstein space in the EGR representation

In the EGR formulation, as we have seen,

$$R_{\text{EGR}} = R - \frac{1}{3}\left(\nabla_e\, J^e + \frac{1}{2}\, J^2\right),$$

and while keeping R constant, we see that when $J^a = const$, we are guaranteed that R_{EGR} is also constant. With this choice, inspection shows that the symmetries of the EGR curvature tensors are identical to the Riemannian ones, and that the EGR second-order curvature tensor $(R_{ab})_{\text{EGR}}$ is now symmetric.

If we wish to define the EGR equivalent to the Einstein space, we must take into account the energy-momentum tensor of the persistent background field, which now reduces to the symmetric expression

$$\frac{1}{\varkappa}\, g_{ab}(R^{cd}R_{cd})_{\text{EGR}}\,. \tag{1.26}$$

In the EGR field equations when substance is absent, this term becomes purely geometric

$$g_{ab}(R^{cd}R_{cd})_{\text{EGR}}\,. \tag{1.27}$$

From the EGR Einstein tensor (1.5), we can then infer the new EGR second-order curvature tensor by grouping all remaining terms into the right hand side of the field equations (1.7), and we find the *symmetric EGR second rank curvature tensor* as

$$(R_{ab})_{\text{EGR}} = g_{ab}\,\lambda_{\text{EGR}} \tag{1.28}$$

with

$$\lambda_{\text{EGR}} = 3\,K_{\text{EGR}} = -\frac{1}{4}\left[\frac{1}{2}\left(R - \frac{1}{6}\,J^2\right) - (R^{cd}R_{cd})_{\text{EGR}}\right], \tag{1.29}$$

where the last term of the right-hand side is assumed to be nearly constant. This equation, (1.28), can be considered as representing the EGR formulation of the classical Einstein space.

This result closely matches our earlier statement where the *prevailing term* $\frac{1}{6}\,g_{ab}\,J^2$ (see [6], formula 3.25) was regarded as generalizing the regular Riemannian term $g_{ab}\lambda$, when the persistent field is discarded.

This derivation allows one to emphasize the arbitrary introduction of the long-debated *cosmological term* λg_{ab} within the Riemannian physics, whereas the EGR theory provides a natural justification for its mere existence.

We will thus simply define the EGR space-time metric of a constant curvature K as

$$(R_{abed})_{\text{EGR}} = K_{\text{EGR}}\big(g_{ae}\,g_{bd} - g_{ad}\,g_{be}\big)\,. \tag{1.30}$$

Chapter 2. Theory of Varied Fields

§2.1. Linear differential operations in the EGR framework

§2.1.1. Definitions

The varied field theory, as put forward by Lichnérowicz [10], relies on the infinitesimal finite variation of the metric tensor g_{ab} which defines a new tensor

$$\delta g_{ab} = h_{ab}\,, \tag{2.1}$$

$$\delta g^{ab} = -g^{ac} g^{bd} h_{cd} = -h^{ab} . \tag{2.2}$$

This fundamentally differs from the regular *linearized gravitation theory* which is based on the slight deviation

$$g_{ab} = \eta_{ab} + h_{ab} , \qquad h_{ab} \ll 1 ,$$

and where h_{ab} is (loosely) regarded as a tensor defined in a *flat background space-time*. By contrast, the variation (2.1) takes place in the chosen manifold, and it determines the varied connections and curvature tensors which retain the same properties as their generic quantities. By doing so, the corresponding finite variations can adequately fit in the quantization process. Before detailing those derivations, we will need first to define some differential operations.

§2.1.2. The generalized EGR Laplacian

On the oriented manifold M of class C^{h+1} (always equipped with the metric g_{ab}), we consider the second-order linear differential operator Δ_{EGR} on the p-tensors, such that

$$(\Delta_{\text{EGR}} T)_{a_1 \ldots a_p} = -\mathrm{D}^b \mathrm{D}_b T_{a_1 \ldots a_p} = -g^{bc} \mathrm{D}_b \mathrm{D}_c T_{a_1 \ldots a_p} . \tag{2.3}$$

This operator transforms any C^{k+2} tensor (where $0 \leqslant k \leqslant h-2$) into a C^k tensor.

Let now d_{EGR} and δ_{EGR} denote, respectively, the EGR exterior differential, and the EGR co-differential operators acting on forms.

The EGR differential operator d_{EGR} is built as the anti-symmetrized EGR covariant derivative and it generalizes the Riemannian curl operator

$$(d_{\text{EGR}} F)_{abc} = \mathrm{D}_a F_{bc} + \mathrm{D}_c F_{ab} + \mathrm{D}_b F_{ca} .$$

The EGR co-differential operator δ_{EGR} generalizes the Riemannian divergence operator

$$(\delta_{\text{EGR}} F)_b = \mathrm{D}^a F_{ab} .$$

In the case of *anti-symmetric tensors* we make use of the EGR Laplacian in the sense of *Georges de Rham*:

$$\Delta_{\text{EGR}} \boldsymbol{T} = (d_{\text{EGR}} \delta_{\text{EGR}} + \delta_{\text{EGR}} d_{\text{EGR}}) \boldsymbol{T} . \tag{2.4}$$

Then Δ_{EGR} commutes with d_{EGR} and δ_{EGR}, since $d^2_{\text{EGR}} = \delta^2_{\text{EGR}} = 0$. Explicitly, the Laplacian Δ_{EGR} is expressed with covariant derivatives as follows

$$(\Delta_{\rm EGR} T)_{a_1\ldots a_p} = -\,\mathrm{D}^c\mathrm{D}_c T_{a_1\ldots a_p} + \frac{1}{(p-1)!}\,\varepsilon^{db_2\ldots b_p}_{a_1\ldots a_p} R_{de} T^{e}_{b_2\ldots b_p} - \\ - \frac{1}{(p-2)!}\,\varepsilon^{deb_2\ldots b_p}_{a_1\ldots a_p} R_{dcef}\, T^{cf}_{b_2\ldots b_p}\,, \tag{2.5}$$

where $\varepsilon^{db_1\ldots b_2}_{a_1\ldots a_p}$ and $\varepsilon^{deb_2\ldots b_p}_{a_1\ldots a_p}$ are the generalized Kronecker tensors which take on the numerical values:

$+1$, if all indices $a_1\ldots a_p$ are distinct, and if the substitution s which makes $b_1\ldots$ to $a_1\ldots$ is pair;

-1, if indices $a_1\ldots a_p$ are all distinct for odd s;

or 0, for all other cases.

Generally speaking, if we denote by C a linear operator field on tensors, and B a linear application field on the same tensors, we define the EGR differential operator

$$\mathrm{L}\boldsymbol{T} = \Delta_{\rm EGR}\boldsymbol{T} + B^b\mathrm{D}_b\boldsymbol{T} + C\boldsymbol{T} \tag{2.6}$$

which transforms any tensor of class C^{k+2} into a tensor of class C^{k}.

The *adjoint* differential operator is thus defined by

$$\mathrm{L}^*\boldsymbol{V} = \Delta_{\rm EGR}\boldsymbol{V} - \mathrm{D}_b B^{*b}\boldsymbol{V} + C^*\boldsymbol{V}$$

with

$$B^{*b} = -B^b, \qquad C^* = C - \mathrm{D}_b B^b.$$

The EGR Laplacian of an arbitrary tensor $\boldsymbol{T}$ defined by (2.5) has the following properties:

a) It is self-adjoint;

b) It commutes with all contractions and with all index transpositions.

Furthermore, if $\boldsymbol{T}$ has zero covariant derivative,

$$\Delta_{\rm EGR}(\boldsymbol{T}\otimes\boldsymbol{V}) = \boldsymbol{T}\otimes\Delta_{\rm EGR}\boldsymbol{V}$$

for any 2-tensor $\boldsymbol{T}$ and vector $\boldsymbol{A}$, we have

$$\delta\Delta_{\rm EGR}\boldsymbol{T} = \Delta_{\rm EGR}\,\delta\boldsymbol{T}, \qquad \mathrm{D}\Delta_{\rm EGR}\boldsymbol{A} = \Delta_{\rm EGR}\mathrm{D}\boldsymbol{A}.$$

Therefore, for an anti-symmetric tensor $\boldsymbol{T}$ of rank 2, we have

$$(\Delta_{\rm EGR} T)_{ab} = -\,\mathrm{D}^c\mathrm{D}_c T_{ab} + (R^d_a)_{\rm EGR}\, T_{db} + \\ + (R^d_b)_{\rm EGR}\, T_{ad} - 2(R_{acbe})_{\rm EGR}\, T^{ce}. \tag{2.7}$$

This last relation will be useful in discussing further results related to symmetric propagators.

§2.2. EGR curvature tensor variations

§2.2.1. The EGR curvature 4th-rank tensor variation

Let us consider the EGR manifold that reduces to the EGR Einstein equations

$$(R_{ab})_{\text{EGR}} = g_{ab}\,\lambda_{\text{EGR}} \tag{2.8}$$

with the following obvious constraint

$$\delta(R_{ab})_{\text{EGR}} = \lambda_{\text{EGR}}\,h_{ab}\,. \tag{2.9}$$

We are now going to evaluate the corresponding variations of the general connection

$$\delta\Gamma^c_{ab} = W^c_{ab} \tag{2.10}$$

so, we first calculate

$$\delta V_{abd} = \frac{1}{2}\left(\partial_a h_{bd} + \partial_b h_{ad} - \partial_d h_{ab}\right), \tag{2.11}$$

$$\delta V_{abd} = \frac{1}{2}\left(\mathrm{D}_a h_{bd} + \mathrm{D}_b h_{ad} - \mathrm{D}_d h_{ab}\right) + h_{de}\,\Gamma^e_{ab}\,, \tag{2.12}$$

hence

$$W^c_{ab} = \delta g^{cd}\,V_{abd} + \frac{1}{2}\left(\mathrm{D}_a h^c_b + \mathrm{D}_b h^c_a - \mathrm{D}^c h_{ab}\right) + h^c_e\,\Gamma^e_{ab}\,, \tag{2.13}$$

that is

$$W^c_{ab} = \frac{1}{2}\left(\mathrm{D}_a h^c_b + \mathrm{D}_b h^c_a - \mathrm{D}^c h_{ab}\right) + h^c_e\,\Gamma^e_{ab} - h^c_d\,\Gamma^d_{ab}\,.$$

Eventually, we find

$$W^c_{ab} = \frac{1}{2}\left(\mathrm{D}_a h^c_b + \mathrm{D}_b h^c_a - \mathrm{D}^c h_{ab}\right). \tag{2.14}$$

Now setting $W_{cab} = g_{cd}\,W^d_{ab}$, we have

$$W_{cab} = \frac{1}{2}\left(\mathrm{D}_a h_{bc} + \mathrm{D}_b h_{ac} - \mathrm{D}_c h_{ab}\right). \tag{2.15}$$

The variation of the EGR tensor components (2.9) expressed with the tensor W_{cab} is then given by

$$\delta(R^a_{bcf})_{\text{EGR}} = \mathrm{D}_c\,W^a_{bf} - \mathrm{D}_f\,W^a_{bc}\,. \tag{2.16}$$

Let us now evaluate the varied tensors

$$\delta(R_{abcf})_{\text{EGR}} = H_{abcf}\,, \qquad \delta(R^{abcf})_{\text{EGR}} = H^{abcf}. \tag{2.17}$$

Since we are here in the EGR Einstein space-time picture, inspection shows that all Riemannian symmetries are also satisfied by the EGR tensor $(R_{abcf})_{\rm EGR}$ and equally hold for H_{abcf}, i.e.

$$H_{abcf} = H_{bacf} = -H_{abfc} = H_{cfab}\,. \tag{2.18}$$

If Ξ denotes the summation after circular permutation on indices, this tensor also satisfies the identity

$$\Xi\, H_{abcf} = 0 \tag{2.19}$$

and the Bianchi identity

$$\Xi\, \mathrm{D}_e H_{abcf} = 0\,. \tag{2.20}$$

Let B_{acbf} be an arbitrary tensor of the 4th rank; we introduce the denotation ${}^0\Sigma$ which acts on B_{acbf} as

$${}^0\Sigma B_{acbf} = B_{acbf} + B_{bfac} - B_{bcaf} - B_{afbc}\,.$$

§2.2.2. Relation of the tensor h_{ab} with R_{abcd}

We first evaluate the tensor

$$M_{abfg}(\underline{h}) = \delta(R_{abfg}) \tag{2.21}$$

which verifies (2.18–2.19), and where $\underline{h} \Leftrightarrow h_{ab}$.

Let us then introduce the symmetric tensor $P_{hked}(\underline{h})$:

$${}^0\Sigma\, \mathrm{D}_d \mathrm{D}_k\, h_{eh} = \mathrm{D}_d \mathrm{D}_k\, h_{he} + \mathrm{D}_h \mathrm{D}_e\, h_{kd} - \mathrm{D}_e \mathrm{D}_d\, h_{hk} - \mathrm{D}_k \mathrm{D}_e\, h_{hd}\,. \tag{2.22}$$

We can show that

$$2M_{abfg}(\underline{h}) = -P_{abfg} - h_{ac}(R^{c\cdot\cdot\cdot}_{\cdot bfg})_{\rm EGR} - h_{bc}(R_{a\cdot fg}^{\cdot c\cdot\cdot})_{\rm EGR} \tag{2.23}$$

so that we can infer the components of another tensor Q_{abfg}

$$Q_{abfg}(\underline{h}) = -P_{abfg} - h_{fc}(R_{ab\cdot g}^{\cdot\cdot c\cdot})_{\rm EGR} - h_{gc}(R_{abf\cdot}^{\cdot\cdot\cdot c})_{\rm EGR} \tag{2.24}$$

uniquely expressed as a function of both the h_{ab} and the EGR curvature tensor $(R_{abcd})_{\rm EGR}$.

After a lengthy tedious calculation, one finds

$$Q_{abfg}(\underline{h}) = M_{abfg}(\underline{h}) + g_{ac}\, g_{bh}\, g_{fk}\, g_{ge}\, \delta(R^{chke})_{\rm EGR}(\underline{h})\,. \tag{2.25}$$

The quantity $Q_{abfg}(\underline{h})$ will play a major role in view of quantizing the EGR gravitational field.

We will also need to evaluate $Q_{abfg}(\mathcal{D}\boldsymbol{A})$ with $\underline{h} = \mathcal{D}\boldsymbol{A}$, where the *extended Lie derivative* of the metric tensor with respect to the arbitrary vector $\boldsymbol{A}$ is given according to Marquet [11]:

$$(\mathcal{D}\boldsymbol{A})_{ab} = A_{b;a} + A_{a;b} + g_{ab}\left(g^{ik}\,\mathrm{D}_c\,g_{ik}\,A^c\right). \tag{2.26}$$

Let us denote by $\mathcal{L}$ the *extended Lie derivative operator* and we write

$$\mathcal{D}\boldsymbol{A} = \mathcal{L}_{A}\,\boldsymbol{g}\,. \tag{2.27}$$

Respectively, $M_{abfg}(\mathcal{D}\boldsymbol{A})$ can be shown to be the EGR Lie derivative of the vector $(R_{abfg})_{\mathrm{EGR}}$ with respect to the vector $\boldsymbol{A}$. We have

$$M_{abfg}(\mathcal{D}\boldsymbol{A}) = \mathcal{L}_{A}(R_{abfg})_{\mathrm{EGR}} = A^d\,\mathrm{D}_d(R_{abfg})_{\mathrm{EGR}} + {}^0\Sigma\,\mathrm{D}_a A_d (R^{d\cdots}_{\cdot bfg})_{\mathrm{EGR}}\,.$$

Hence, for the tensor Q_{abfg} (2.25), we have

$$Q_{abfg}(\mathcal{D}\boldsymbol{A}) = 2A^d\,\mathrm{D}_d(R_{abfg})_{\mathrm{EGR}} + {}^0\Sigma\,(\mathrm{D}_a A_d - \mathrm{D}_d A_a)(R^{d\cdots}_{\cdot bfg})_{\mathrm{EGR}} \tag{2.28}$$

and after a further tedious calculation, we obtain

$$\mathrm{D}_k\,Q_{abfg}(\underline{h}) = 2\,\delta\mathrm{D}_k(R_{abfg})_{\mathrm{EGR}} + \\ + {}^0\Sigma\,(\mathrm{D}_a\,h_{dk} - \mathrm{D}_d\,h_{ak})(R^{d\cdots}_{\cdot bfg})_{\mathrm{EGR}} - {}^0\Sigma\,h_{ad}\,\mathrm{D}_k\,(R^{d\cdots}_{\cdot bfg})_{\mathrm{EGR}}\,. \tag{2.29}$$

§2.2.3. Second-order curvature tensor variation

The relevant variation of the EGR tensor $(R_{ab})_{\mathrm{EGR}}$ is

$$\delta(R_{bf})_{\mathrm{EGR}} = \mathrm{D}_d\,W^d_{bf} - \mathrm{D}_f\,W^d_{db}\,. \tag{2.30}$$

Taking account of (2.15), one may write

$$2\,\delta(R_{ab})_{\mathrm{EGR}} = g^{de}\,\mathrm{D}_d\left(\mathrm{D}_a\,h_{be} + \mathrm{D}_b\,h_{ae} - \mathrm{D}_e\,h_{ab}\right) - \mathrm{D}_a\,\mathrm{D}_b\,h\,, \tag{2.31}$$

where we set

$$h = g^{de}\,h_{de}\,.$$

Considering the *Ricci identity* within the EGR framework, applied to the tensor h^{ab}

$$h^e_{b',ea} - h^e_{b',ae} = (R_{ad})_{\mathrm{EGR}}\,h^d_b - h^{ed}(R_{aebd})_{\mathrm{EGR}}\,,$$

one deduces for (2.31):

$$2\,\delta(R_{ab})_{\mathrm{EGR}} = -\,\mathrm{D}^e\,\mathrm{D}_e\,h_{ab} + (R^d_a)_{\mathrm{EGR}}\,h_{db} + (R^d_b)_{\mathrm{EGR}}\,h_{ad} - \\ -\,2\,(R_{aebd})_{\mathrm{EGR}}\,h^{ed} + \left(\mathrm{D}_a\,\mathrm{D}_e\,h^e_b + \mathrm{D}_b\,\mathrm{D}_e\,h^e_a - \mathrm{D}_a\,\mathrm{D}_b\,h\right). \tag{2.32}$$

And, together with formula (2.26), this leads to

$$2\,\delta(R_{ab})_{\text{EGR}} = \Delta_{\text{EGR}}\, h_{ab} + (\mathcal{D}\boldsymbol{k})_{ab}\,, \tag{2.33}$$

where the vector $\boldsymbol{k}(\underline{h})$ has components

$$k_a(\underline{h}) = \mathrm{D}_d\, h^d_a - \frac{1}{2}\,\mathrm{D}_a\, h\,. \tag{2.34}$$

From (2.33), the contraction yields

$$g^{de}\,\delta(R_{de})_{\text{EGR}} = \frac{1}{2}\,\Delta_{\text{EGR}}\, h + \mathrm{D}_a\, k^a(\underline{h})\,. \tag{2.35}$$

For the *EGR Einstein space* (1.28), eventually holds the following relation

$$\mathrm{D}^a\left[\delta(R_{ab})_{\text{EGR}} - \frac{1}{2}\, g_{ab}\, g^{ed}\,\delta(R_{ed})_{\text{EGR}}\right] = \lambda_{\text{EGR}}\, k_b(\underline{h}) \tag{2.36}$$

which could be formally derived from the variation of the conservation identity of the EGR Einstein tensor (1.5) reduced here to its *symmetric* version. This is an important result, as (2.36) precisely matches the equivalent Riemannian relation derived by Lichnérowicz.

Such an equivalence lends strong support to the EGR theory, thus appearing as a legitimate generalization of the classical General Relativity in the varied field formulation.

With the EGR *symmetric* second-order curvature tensor variation still being bound to the condition

$$\delta(R_{ab})_{\text{EGR}} = \lambda_{\text{EGR}}\, h_{ab}\,, \tag{2.37}$$

inspection shows that this equation is invariant upon the EGR gauge transformation

$$\underline{h}' \to \underline{h} + \mathcal{D}\boldsymbol{A}\,, \tag{2.38}$$

where $\boldsymbol{A}$ is, as usual, an arbitrary infinitesimal vector. This is certainly true, provided the vector J_a is constant, which is indeed the case according to (2.37).

Lemma (Lichnérowicz)

For the EGR Einstein space, we have

$$(\,\Delta_{\text{EGR}} \;-\; 2\,\lambda_{\text{EGR}}\,)\, A_b = -\,(\,\mathrm{D}^a\,\mathrm{D}_a\, A_b + \lambda_{\text{EGR}}\, A_b\,)\,.$$

As a result of Lichnérowicz' lemma, a formal calculation leads to

$$(\,\Delta_{\text{EGR}} - 2\,\lambda_{\text{EGR}}\,)\,\boldsymbol{A} = \boldsymbol{k}(\underline{h})$$

so, with the constraint $\boldsymbol{k}(\underline{h}) = 0$ which is the initial condition, we have

$$(\Delta_{\text{EGR}} - 2\lambda_{\text{EGR}})\,\boldsymbol{A} = 0\,,$$

and we then eventually obtain the field equations for $\underline{h}$ which take the form

$$(\Delta_{\text{EGR}} - 2\lambda_{\text{EGR}})\,\underline{h} = 0\,. \tag{2.39}$$

Chapter 3. Quantizing Varied Fields

§3.1. Tensor propagators

§3.1.1. Displacement bi-tensors

Tensor propagators have been introduced in order to generalize the scalar propagator on a curved manifold. Indeed, in an Euclidian space, the quantum field theory makes an intensive use of Fourier's transform. In a curved space-time, this transform no longer applies and therefore an alternate theory developed by Lichnérowicz, can be adequately substituted, which is based on the so-called concept of *displacement bi-tensors*.

On the differentiable manifold M, we consider a point x' located in the neighbourhood of another point x. Along the EGR geodesic connecting x' to x, can be defined a *displacement* which represents a *canonical isomorphism* (base-independence) of the space T_x at x tangent to the manifold onto the tangent space $\mathrm{T}_{x'}$ at x'.

The free bases $e_a(x)$ and $e_{c'}(x')$ are attached to those neighbourhoods. The relevant isomorphism therefore defines a *bi-tensor* denoted by $\boldsymbol{t}$ which is named *displacement tensor* and whose components are labeled $t_a^{c'}$.

For further analysis and subsequent properties, it is useful to refer to our earlier publication [12].

In the foregoing, we will restrict our study to *massless fields* only.

§3.1.2. Elementary kernels and propagators

In the most general manner, the definition of any commutator requires the analytic description of the isotropic EGR conoids (see §1.1.2).

For this, always on the manifold M, we denote by $(\mathrm{C}_{x'})_{\text{EGR}}$ the characteristic EGR conoid with apex x' and wherefrom are generated the EGR geodesics.

This regular point x' belongs to the compact subset Ω, *neighbourhood homeomorphic* to the *Euclidean open ball*, that is, the tangent vector space T_x at x'.

Herein the subset Ω exhibits three regions: *future* I^{+} of x', *past* I^{-} of x, and *elsewhere*. The first two regions characterize *two* temporal domains $(\mathrm{C}^{\pm}_{x'})_{\mathrm{EGR}}$, (compact sets), which correspond to the subdivision of $(\mathrm{C}_{x'})_{\mathrm{EGR}}$ in two half conoids, one oriented towards the future, the other towards the past.

With the following considerations being purely *local*, it can be shown that there exist two p-tensors satisfying

$$\mathrm{L}^{*}_{x}\boldsymbol{E}^{(p)^{\pm}}(x,x') = \delta^{(p)}(x,x')\,. \tag{3.1}$$

The $\boldsymbol{E}^{(p)^{\pm}}(x,x')$ are two elementary solutions called *elementary kernels* of L on $\Omega\times\Omega$, and which, for each x', do have their supports respectively in $\mathrm{I}^{+}(x')$ and $\mathrm{I}^{-}(x')$.

One may then define in $(\mathrm{C}_{x'})_{\mathrm{EGR}}$ the *EGR p-tensor*

$$\boldsymbol{E}^{(p)}(x,x') = \boldsymbol{E}^{(p)^{+}}(x,x') - \boldsymbol{E}^{(p)^{-}}(x,x') \tag{3.2}$$

which is by definition the tensor propagator associated with the operator L. In the Minkowski space, the scalar propagator $\boldsymbol{E}^{(0)}$ reduces to the *Jordan-Pauli propagator* denoted by $\boldsymbol{D}$.

§3.1.3. Propagators associated with the operator $\boldsymbol{\Delta}_{\mathrm{EGR}}+\boldsymbol{\mu}$

Letting μ be a constant, the operator $\Delta_{\mathrm{EGR}}+\mu$ acts on anti-symmetric tensors of rank p.

Anti-symmetrizing the *kernels* $\boldsymbol{E}^{(p)^{\pm}}$, we obtain *two* unique solutions $\boldsymbol{G}^{(p)^{\pm}}$ (p-forms), which satisfy for each x' and x, the partial derivative equations

$$\big[(\Delta_x)_{\mathrm{EGR}}+\mu\big]\,\boldsymbol{G}^{(p)^{\pm}}(x,x') = \delta^{(p)}(x,x')\,, \qquad p=0,1,\ldots,n$$

with support respectively *in* and *on* $\mathrm{C}^{+}_{x'}$ and $\mathrm{C}^{-}_{x'}$.

Likewise, for each x, these kernels define *two* solutions near x' within Ω

$$\big[(\Delta_{x'})_{\mathrm{EGR}}+\mu\big]\,\boldsymbol{G}^{(p)^{\pm}}(x',x) = \delta_{(p)}(x',x)\,.$$

The difference

$$\boldsymbol{G}^{(p)}(x,x') = \boldsymbol{G}^{(p)^{+}}(x',x) - \boldsymbol{G}^{(p)^{-}}(x',x) \tag{3.3}$$

defines the *anti-symmetric propagator* associated with the operator $\Delta_{\mathrm{EGR}}+\mu$, which is a solution of

$$\big[(\Delta_x)_{\mathrm{EGR}}+\mu\big]\,\boldsymbol{G}^{(p)}(x,x') = 0\,. \tag{3.4}$$

By symmetrizing the elementary kernels $\boldsymbol{E}^{(2)^{\pm}}(x', x)$ (limited to order 2) related to our operator, one notes the emergence of *two symmetric kernels* $\boldsymbol{K}^{\pm}(x', x)$ which are symmetric 2-tensors satisfying at x

$$\left[(\Delta_x)_{\mathrm{EGR}} + \mu\right] \boldsymbol{K}^{\pm}(x', x) = \boldsymbol{t}\,\delta(x, x') \tag{3.5}$$

with $\boldsymbol{t}$ having the components

$$t_{abc'd'} = t_{ac'}\,t_{bd'} + t_{ad'}\,t_{bc'}\,, \tag{3.6}$$

which means that the symmetrization operation was applied to the 2-tensor. We therefore call the *symmetric propagator* related to $\Delta_{\mathrm{EGR}} + \mu$ the symmetric 2-tensor defined by

$$\boldsymbol{K}(x, x') = \boldsymbol{K}^{+}(x', x) - \boldsymbol{K}^{-}(x', x)\,. \tag{3.7}$$

§3.2. Commutation rules

§3.2.1. Electromagnetic field in the Minkowski space

The potential 1-form $\boldsymbol{A}$ induces an electromagnetic field $\boldsymbol{F}$ according to the equations

$$\boldsymbol{F} = d\boldsymbol{A}\,, \qquad d\boldsymbol{A} = A_b \wedge dx^b. \tag{3.8}$$

which are invariant under the gauge transformation

$$A_b \;\longrightarrow\; A'_b = A_b + \partial_b U\,.$$

Classically, with our notations used so far, we express the *commutator* for the *potential* in the form (see [13], formula 11.27)

$$\left[\boldsymbol{A}(x), \boldsymbol{A}(x')\right] = -\frac{\hbar}{i}\left\{\boldsymbol{t}\boldsymbol{D}(x, x')\right\}, \tag{3.9}$$

where $\hbar = \frac{h}{2\pi}$, the mass term is characterized by $\mu = 0$, and the Jordan-Pauli propagator $\boldsymbol{D}$ is related to the regular Laplace operator which satisfies the following conditions

$$\Delta A = 0\,, \qquad \delta A = 0\,. \tag{3.10}$$

Taking this result into account, the commutator (3.9) is written

$$\left[\boldsymbol{A}(x), \boldsymbol{A}(x')\right] = -\frac{\hbar}{i}\left\{\boldsymbol{G}^{(1)}(x, x')\right\} \tag{3.11}$$

which leads, for the electromagnetic field $\boldsymbol{F}$, to the commutator

$$\left[\boldsymbol{F}(x), \boldsymbol{F}(x')\right] = -\frac{\hbar}{i}\left\{d_x d_{x'}\, G^{(1)}(x, x')\right\}. \tag{3.12}$$

Short inspection shows that this commutator is compatible with the regular Maxwell equations

$$d\boldsymbol{F} = 0\,, \qquad \delta\boldsymbol{F} = 0\,, \tag{3.13}$$

and $\delta\boldsymbol{A} = 0$ once some initial conditions have been applied.

§3.2.2. Commutator for the varied EGR second-order curvature tensor

By a strict analogy, we have the evident correspondence

$$\Delta_{\text{EGR}}\,\underline{h} = 0 \quad\longrightarrow\quad \boldsymbol{A} = 0\,,$$
$$\boldsymbol{k}(\underline{h}) = 0 \quad\longrightarrow\quad \delta\boldsymbol{A} = 0\,,$$

so, we are led to adopt the commutator for $\underline{h}$

$$\big[\underline{h}(x),\,\underline{h}(x')\big] = \frac{\mathfrak{G}\hbar}{i\,c^2}\left\{\boldsymbol{K}(x,x') - \boldsymbol{g}(x)\,\boldsymbol{g}(x')\,\boldsymbol{G}^{(0)}(x,x')\right\}, \tag{3.14}$$

where the propagators are related to the operator $(\Delta_{\text{EGR}} - 2\,\lambda_{\text{EGR}})$, and $\boldsymbol{g} = g_{ab}\,dx^a \otimes dx^b$.

§3.3. Quantization in the constant-curvature space

§3.3.1. Commutator for higher-order fields

In the Minkowski space with metric tensor η_{ab}, we use here a system of orthonormal basis. It is interesting to evaluate the commutator (3.14) as applied to the field $\underline{h}$, for the tensor H_{abcd} (2.17) which verifies the equations (2.18–2.19).

The commutator for H_{abcd} is classically given by

$$\begin{aligned}&\big[H_{abcd}(x), H_{efgh}(x')\big] = \\ &= \frac{\mathfrak{G}\hbar}{4\,i\,c^2}\Big\{\big({}^0\Sigma\,\eta_{bf}\,\partial_e\partial_a\big)\big({}^0\Sigma\,\eta_{dh}\,\partial_g\partial_e\big) + \big({}^0\Sigma\,\eta_{df}\,\partial_e\partial_c\big)\big({}^0\Sigma\,\eta_{bh}\,\partial_g\partial_a\big) - \\ &- \big({}^0\Sigma\,\eta_{fh}\,\partial_e\partial_g\big)\big({}^0\Sigma\,\eta_{bd}\,\partial_c\partial_a\big)\Big\}\,\boldsymbol{D}_{(0)}(x,x')\,.\end{aligned} \tag{3.15}$$

In an arbitrary basis system and after changing the indices, a lengthy calculation shows that the term in the brackets can be split up into two following parts. The first part is

$${}^0\Sigma\,\mathrm{D}_{g'}\,\mathrm{D}_{e'}\,{}^0\Sigma\,\mathrm{D}_c\,\mathrm{D}_a\left[t_{bf'}\,t_{dh'} + t_{df'}\,t_{bh'}\right]\boldsymbol{D}_{(0)}\,, \tag{3.16}$$

i.e.

$$\mathrm{Q}(x')\,\mathrm{Q}(x)\,\boldsymbol{K}(x,x')\,, \tag{3.17}$$

where Q_x is the operator Q defined above, acting on the tensors of rank 2 and which is defined at the point x.

The second part is

$$^{0}\Sigma \mathrm{D}_{g'} \mathrm{D}_{e'} \, ^{0}\Sigma \mathrm{D}_c \mathrm{D}_a \left(g_{f'h'} \, g_{bd}\right) \boldsymbol{D}_{(0)}(x, x')\,, \tag{3.18}$$

i.e.

$$\mathrm{Q}_{x'} \, \mathrm{Q}_x \, \boldsymbol{g}(x) \boldsymbol{g}(x') \boldsymbol{D}_{(0)}\,. \tag{3.19}$$

Eventually, we obtain for an arbitrary basis

$$\left[\boldsymbol{H}(x), \boldsymbol{H}(x')\right] = \frac{\mathfrak{G}\hbar}{4ic^2} \, \mathrm{Q}_{x'} \, \mathrm{Q}_x \left\{\boldsymbol{K}(x, x') - \boldsymbol{g}(x) \boldsymbol{g}(x') \boldsymbol{D}_{(0)}(x, x')\right\}, \tag{3.20}$$

where $\boldsymbol{D}_{(0)}$ and $\boldsymbol{K}$ are respectively the scalar and symmetric propagators of rank 2 associated with the operator Δ_{EGR}.

§3.3.2. The EGR constant-curvature space

We now consider a curved manifold specialized to the EGR space with a constant curvature as defined in (1.30), in which case we make use of the results of §2.2.

From the derived relation (2.28), one infers

$$\mathrm{Q}(\mathcal{D}\boldsymbol{A}) = 0 \tag{3.21}$$

for any vector $\boldsymbol{A}$. Moreover, from (2.29), for any symmetric tensor $\underline{h}$, we have

$$\Xi \mathrm{D}_k \, Q_{abfg}(\underline{h}) = 0\,. \tag{3.22}$$

Consider the commutator (3.14): using the operators Q_x and $\mathrm{Q}_{x'}$ and taking account of (3.21), we get

$$\left[\mathrm{Q}\,\underline{h}(x), \mathrm{Q}\,\underline{h}(x')\right] = \frac{\mathfrak{G}\hbar}{ic^2} \, \mathrm{Q}_x \, \mathrm{Q}_{x'} \left\{\boldsymbol{K}(x, x') - \boldsymbol{g}(x) \boldsymbol{g}(x') \boldsymbol{G}^{(0)}\right\}. \tag{3.23}$$

Setting

$$H_{abfg}(\underline{h}) = \frac{1}{2} \, Q_{abfg}(\underline{h})$$

which has the same properties, we obtain the EGR commutator

$$\left[\boldsymbol{H}(x), \boldsymbol{H}(x')\right] = \frac{\mathfrak{G}\hbar}{4ic^2} \, \mathrm{Q}_x \, \mathrm{Q}_{x'} \left\{\boldsymbol{K}(x, x') - \boldsymbol{g}(x) \boldsymbol{g}(x') \boldsymbol{G}^{(0)}\right\}, \tag{3.24}$$

which is formally the extension of the commutator (3.20) established for any arbitrary basis, in the Minkowski space.

Thus, the theory elaborated for the Minkowski space has been successfully generalized to the EGR constant-curvature space.

Conclusion. In the previous exposition, we have only sketched the full theory of Lichnérowicz which actually thoroughly covers the massive field commutators among which the *Fierz commutators* (spin 2 fields) are formally generalized to the Riemannian Einstein spaces.

Generally speaking, the definition of commutators leads to a physical description of the quantized *varied* gravitational field represented by a 4th-rank tensor.

The important work of Lichnérowicz has proven essential for the initial knowledge of this Riemannian quantization technique even if it is restricted to a constant-curvature space.

Performing a similar derivation within the extended Einstein space explicitly shows that the EGR field 4th-rank tensor, when *varied*, fits in the same quantization pattern.

In addition, the EGR Einstein space necessarily implies the natural existence of a generalized cosmological constant which is arbitrarily introduced in the Riemannian framework.

This natural constant, however, remains a particular case, since in the EGR theory, such a cosmological term is variable as it is intrinsically part of the relevant geometry inherent to the theory.

All these tend once more to confirm that the extended General Relativity — the EGR theory suggested in [6] — is a viable model that offers and justifies broad new perspectives in physics.

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