On the Physical Nature of the Wave Function: A New Approach through the EGR Theory

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Abstract: In this paper, we first recall the quantum theory of Louis de Broglie which attempted to give the usual wave function a real and truly physical nature, and which is closely associated to a massive particle. The resulting Double Solution Theory is then interpreted in terms of a physical fluid described in the framework of the Extended General Relativity Theory (EGR theory) approximation. This approach may provide an explanation to the problem arising from the "hidden" medium as set forth by the initial theory.

Contents:

Notations:

4-tensor or 4-vector: small Latin indices $a, b, \ldots = 1, 2, 3, 4$;

3-tensor or 3-vector: small Greek indices $\alpha, \beta, \ldots = 1, 2, 3$;

Ordinary derivative: $\partial_a U$;

Covariant derivative: ∇_a in GR, and D_a in EGR;

Kronecker symbol: $\delta_{ab} = (+1 \text{ if } a = b; \; 0 \text{ if } a \neq b)$;

Three-dimensional vectorial quantities: $\boldsymbol{P} = P_\alpha$.

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Introduction

The original wave function first discovered by Louis de Broglie [1] in his famous Wave Mechanics Theory is always acknowledged as a statistical entity. Its physical meaning was almost totally denied in all subsequent quantum field developments despite the Davisson and Germer experiment, which actually detected the wave through diffraction of electrons by a nickel crystal lattice.

This problem dates back from the Solvay Symposium of 1927 in Brussels, when most of the physicists decided to adopt the so-called Copenhagen School Concept of considering quantum physics on pure statistical grounds. Throughout the remainder of his life, de Broglie yet could not believe observable physical phenomena to only follow from abstract mathematical wave functions.

In the late 1960's, he improved his first theory called the *double solution interpretation of Quantum Mechanics*, which describes a particle as closely related to its physical wave and constantly in phase with it. The theory is extremely simple and elegant, but to remain consistent, it requires two constraints:

- The guided particle should permanently exchange energy and momentum from an external (unknown) medium which he named "hidden thermostat";
- In addition, the considered particle should also undergo small energetic random perturbations.

In the past decades, many interesting theories have been provided for explaining the nature of this "sub-quantum" medium, which is assumed to exchange energy and momentum at the quantum level.

In this paper, we suggest to identify this "energy background" with the persistent field of the EGR theory. The hydrodynamic interpretation for the particle's probability density as depicted by de Broglie, is here given with physical consistency.

Chapter 1. Interpretation of Wave Mechanics by Means of Louis de Broglie's Theory of Double Solution

§1.1. The reasons for implementing the theory

For almost a century, the wave-particle duality first discovered by Einstein, in his theory of light quanta, has been the basis of present day Quantum Physics. As an essential contribution, the wave mechanics theory of Louis de Broglie has successfully extended this duality to all known particles. Shortly after, de Broglie further developed the Double Solution Theory based on two striking observations.

In the framework of the Special Theory of Relativity, it is noticed that the frequency ν_0 of a plane monochromatic wave is transformed as

$$\nu_c = \nu_0 \sqrt{1-\beta^2}\,,$$

while a clock's frequency ν_0 is transformed according to

$$\nu_c = \frac{\nu_0}{\sqrt{1-\beta^2}}$$

with the phase velocity

$$\widetilde{v} = \frac{c}{\beta} = \frac{c^2}{v}\,.$$

It is noticed that the four-vector defined by the gradient of the plane monochromatic wave can be linked to the energy-momentum four-vector of a particle by introducing Planck's constant h, thus writing

$$W = h\nu\,, \qquad \lambda = \frac{h}{p}\,, \tag{1.1}$$

where p is the particle's momentum and λ is its wavelength.

If the particle is considered as containing the rest energy

$$M_0\, c^2 = h\nu_0\,,$$

we may compare it to a small clock of a frequency ν_0 so that when moving with a velocity $v=\beta c$, its frequency, different from that of the wave, is then

$$\nu_c = \nu_0 \sqrt{1-\beta^2}\,.$$

It can be further shown that the particle has an internal vibration which is constantly in phase with the vibration of the surrounding wave.

In the spirit of the theory, the wave is regarded as a physical entity having a very small amplitude which cannot be arbitrarily normed, and which is distinct from the wave ψ having a statistical significance in the usual quantum mechanical formalism.

Let us call ϑ the physical wave which is connected to the wave ψ by the relation $\psi = C\vartheta$, where C is a normalizing factor. The wave ψ has then the nature of a subjective probability representation formulated by means of the objective wave ϑ. Therefore, the wave mechanics is complemented by the Double Solution Theory, so ψ and ϑ are two solutions of the same equation.

If the complete solution of the equation representing the wave ϑ, or, if preferred, the wave ψ (since both waves are equivalent according to $\psi = C\vartheta$) is written as

$$\vartheta = a(x,y,z,t)\,\exp\left[\frac{i}{\hbar}\,\phi(x,y,z,t)\right], \qquad \hbar = \frac{h}{2\pi}\,, \tag{1.2}$$

where a and ϕ are real functions, the energy W and momentum vector $\boldsymbol{p}$ of the particle localized at point x, y, z at time t are given by

$$W = \partial_t \phi\,, \qquad \boldsymbol{p} = -\,\mathrm{grad}\,\phi\,, \tag{1.3}$$

which in the case of a plane monochromatic wave, where one has

$$\phi = h\left[\nu t - \frac{(\alpha x + \beta y + \gamma z)}{\lambda}\right],$$

yields equation (1.1) for W and $\boldsymbol{p}$.

§1.2. The guidance formula and the quantum potential

Taking Schrödinger's equation for the scalar wave phase ϑ in the external potential U, we get

$$\partial_t \vartheta = \frac{\hbar}{2im}\,\Delta\vartheta + \frac{i}{\hbar}\,U\vartheta\,. \tag{1.4}$$

This complex equation suggests that ϑ should be represented by two real functions linked by two real equations, leading to

$$\vartheta = a\,\exp\frac{i\phi}{\hbar}\,, \tag{1.5}$$

where a (the wave's amplitude) and ϕ (its phase) are both real values. Taking this value into equation (1.4), we arrive at two important equations

$$\partial_t \phi - U - \frac{1}{2m}\,(\mathrm{grad}\,\phi)^2 = \frac{\hbar}{2m}\,\frac{\Delta a}{a}\,, \tag{1.6}$$

$$\partial_t a^2 - \frac{1}{m}\,\mathrm{div}\big(a^2\,\mathrm{grad}\,\phi\big) = 0\,. \tag{1.7}$$

If terms involving Planck's constant $\hbar$ in equation (1.6) are neglected (which amounts to disregarding quanta), and if we set $\phi = S$, this equation becomes

$$\partial_t S - U = \frac{1}{2m}\,(\mathrm{grad}\,S)^2\,. \tag{1.8}$$

As S is the Jacobi function, equation (1.8) is the Jacobi equation of Classical Mechanics. Only the term containing $\hbar^2$ is responsible for the particle's motion being different from the classical motion.

It can be interpreted as another potential $\mathcal{Q}$, distinct from the classical potential U,

$$\mathcal{Q} = -\frac{\hbar^2}{2m}\,\frac{\Delta a}{a}\,. \tag{1.9}$$

One has thus a variable proper mass

$$M_0 = m_0 + \frac{\mathcal{Q}_0}{c^2}\,,$$

where, in the particle's rest frame, $\mathcal{Q}_0$ is a positive or negative variation of the rest mass, and it represents the "quantum potential" which causes the wave function's amplitude to vary.

By analogy with the classical formulae $\partial_t S = E$ and $\boldsymbol{p} = -\operatorname{grad} S$, with E and $\boldsymbol{p}$ being the classical energy and momentum vector, one may write

$$\partial_t \phi = E\,, \qquad \boldsymbol{p} = -\operatorname{grad}\,\phi\,. \tag{1.10}$$

As in non-relativistic mechanics, where $\boldsymbol{p}$ is expressed as a function of velocity by the relation $\boldsymbol{p} = m\mathbf{v}$, one eventually finds the following result

$$\mathbf{v} = \frac{\boldsymbol{p}}{m} = -\frac{1}{m}\operatorname{grad}\phi\,, \tag{1.11}$$

which is the *guidance formula*; it gives the particle's velocity, at point x, y, z and time t as a function of the local phase variation at this point. Inspection shows that relativistic dynamics applied to the variable proper mass M_0 eventually leads to the following result

$$W = \frac{M_0 c^2}{\sqrt{1-\beta^2}} = M_0 c^2 \sqrt{1-\beta^2} + \frac{M_0 v^2}{\sqrt{1-\beta^2}}\,, \tag{1.12}$$

known as the *Planck-Laue formula*.

In the relativistic form of the theory, equation (1.4) is replaced by the Klein-Gordon equation applied to the wave function ϑ

$$\Box\vartheta - \frac{2i}{\hbar}\,\frac{eV}{c^2}\,\partial_t\vartheta + \frac{2i}{\hbar}\,\frac{e}{c}\sum_{xyz} A_x\,\partial_x\vartheta + \\ + \frac{1}{\hbar^2}\left[m_0^2c^2 - \frac{e^2}{c^2}\left(V^2 - \boldsymbol{A}^2\right)\right]\vartheta = 0\,, \tag{1.13}$$

where the particle's charge e is acted upon by an electromagnetic field with a scalar potential $V(x,\,y,\,z,\,t)$ and a vector potential $\boldsymbol{A}(x,\,y,\,z,\,t)$. Note that the d'Alembertian, as usual, is

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\,.$$

Inserting equation (1.5) into equation (1.13) yields the generalized Jacobi equation (1.14) as well as another continuity equation (1.15)

$$\frac{1}{c^2}\left(\partial_t\phi - eV\right)^2 - \sum_{xyz}\left(\partial_t\phi + \frac{e}{c}A_x\right)^2 = m_0^2c^2 + \hbar^2\,\frac{\Box\, a}{a} = M_0^2c^2, \quad (1.14)$$

$$\frac{1}{c^2}\left(\partial_t\phi - eV\right)\partial_t a - \sum_{xyz}\left(\partial_x\phi + \frac{e}{c}A_x\right)\partial_x a + \frac{a}{2}\,\Box\phi = 0\,, \qquad (1.15)$$

where on the right hand side of (1.14) we have introduced a variable proper mass M_0 defined by

$$M_0 = \sqrt{m_0^2 + \frac{\hbar^2}{c^2}\,\frac{\Box\, a}{a}}\;. \qquad (1.16)$$

Neglecting the terms in $\hbar^2$, equation (1.14) leads to

$$\frac{1}{c^2}\left(\partial_t S - eV\right)^2 - \sum_{xyz}\left(\partial_x S + e\,A_x\right)^2 = m_0^2c^2, \qquad (1.17)$$

which is the Jacobi equation for a charged particle moving in an electromagnetic field with scalar and vector potentials V and $\boldsymbol{A}$, and considered in the framework of relativistic mechanics without quanta.

Keeping the terms in $\hbar^2$ and considering the proper mass M_0 as defined in equation (1.16), one gets

$$\frac{M_0\,c^2}{\sqrt{1-\beta^2}} = \partial_t\phi - eV, \qquad \frac{M_0\,\mathbf{v}}{\sqrt{1-\beta^2}} = -\left(\operatorname{grad}\phi + e\boldsymbol{A}\right), \qquad (1.18)$$

thus, with $\beta = \frac{\mathrm{v}}{c}$, we find the relativistic guidance formula

$$\mathbf{v} = -\frac{c^2\left(\operatorname{grad}\phi + e\boldsymbol{A}\right)}{\partial_t\phi - eV}\;. \qquad (1.19)$$

For the Newtonian approximation with $\boldsymbol{A} = 0$ and $\partial_t\phi - eV \cong m_0c^2$, equation (1.11) is found again.

Here, the quantum force results from the variation of M_0c^2 as the particle moves. For a monochromatic wave, the quantum potential is constantly zero, and one simply has

$$\mathcal{Q} = M_0\,c^2 - m_0\,c^2. \qquad (1.20)$$

In the non-relativistic approximation, $c\to\infty$ and $\Box\, a \cong -\Delta a$. Therefore we obtain

$$\mathcal{Q} = \sqrt{\frac{m_0^2c^4 + c^2\hbar^2\,\Box\, a}{a}} \cong -\frac{\hbar^2}{2m_0}\,\frac{\Delta a}{a}\;. \qquad (1.21)$$

§1.3. Particles with internal vibration and the hidden thermodynamics

The idea of considering the particle as a small clock is of central importance here. Let us look at the self-energy M_0c^2 as the hidden heat content of a particle. One easily conceives that such a small clock has (in its proper system) an internal periodic energy of agitation which does not contribute to the momentum of the whole. This energy is similar to that of a heat-containing body in a thermal equilibrium. Let Q_0 be the heat content of the particle in its own (resting) frame, and viewed in a frame where it has a velocity βc. The contained heat in the second frame will be

$$Q = Q_0 \sqrt{1-\beta^2} = M_0 c^2 \sqrt{1-\beta^2} = h\nu_0 \sqrt{1-\beta^2}\ . \tag{1.22}$$

The particle thus appears at the same time as being a small clock of a frequency

$$\nu = \nu_0 \sqrt{1-\beta^2}$$

and a small reservoir of a heat

$$Q = Q_0 \sqrt{1-\beta^2}$$

moving with the velocity βc.

If ϕ is the phase of the wave $a \exp \frac{i\phi}{\hbar}$, where a and ϕ are real, the guidance theory states that

$$\partial_t \phi = \frac{M_0 c^2}{\sqrt{1-\beta^2}}\,, \qquad -\operatorname{grad}\phi = \frac{M_0 \mathbf{v}}{\sqrt{1-\beta^2}}\,, \tag{1.23}$$

so the Planck-Laue equation can be written as

$$Q = M_0 c^2 \sqrt{1-\beta^2} = \frac{M_0 c^2}{\sqrt{1-\beta^2}} - \boldsymbol{v}\boldsymbol{p}\,. \tag{1.24}$$

Combining (1.23) and (1.24) results in

$$M_0 c^2 \sqrt{1-\beta^2} = \partial_t \phi + \mathbf{v} \operatorname{grad}\phi = \frac{d\phi}{dt}\,.$$

Since the particle is regarded as a clock of a proper frequency $\frac{M_0c^2}{h}$, the phase of its internal vibration expressed by $a_i \exp \frac{i\phi_i}{\hbar}$, with a_i and ϕ_i real values, will be

$$\phi_i = h\nu_0 \sqrt{1-\beta^2}\ t = M_0 c^2 \sqrt{1-\beta^2}\ t\,,$$

thus

$$d\,(\phi_i - \phi) = 0\,. \tag{1.25}$$

The fundamental result agrees with the assumption according to which a particle, as it moves in its own wave, remains constantly in phase with it.

§1.4. The equations of continuity

The equations of continuity are (1.7) and (1.15). First we revert to equation (1.7)

$$\partial_t a^2 - \frac{1}{m}\,\mathrm{div}\big(a^2\,\mathrm{grad}\,\phi\big) = 0\,.$$

Making use of the guidance law (1.11) and setting $\rho = Ka^2$, where K is a constant, equation (1.7) becomes

$$\partial_t \rho + \mathrm{div}\,(\rho \boldsymbol{v}) = 0\,. \tag{1.26}$$

In hydrodynamics, this equation represents the continuity equation. The quantity $\rho d\tau$ is the number of the fluid's molecules in the volume element $d\tau$ moved with the velocity $\boldsymbol{v}$.

We denote by $\frac{D}{dt}$ the derivative taken along the direction of motion of the molecules. The expression

$$\frac{D\,(\rho\, d\tau)}{dt} = 0$$

then expresses the conservation of the fluid.

With a normalization factor, $\rho\, d\tau = a^2\,(x,\, y,\, z,\, t)\, d\tau$ is here assumed to be the probability of finding a single particle at time t in the volume element $d\tau$, at $x,\, y,\, z$.

As the statistical wave ψ solution of the linear equation is purely virtual, it can be defined as everywhere proportional to the real wave ϑ, and so we may set $\psi = C\,\vartheta$, where C is the normalization factor chosen so as to satisfy

$$\int |\psi|^2\, d\tau = 1\,.$$

Chapter 2. The Random Perturbation in the Framework of the EGR Theory

§2.1. The physical requirement

With the simple hydrodynamic picture (1.26), and with the constant K taken to be 1, it is assumed that $a^2\,(x^a) = \rho$ multiplied by $d\tau$ gives, with a normalizing factor, the probability of finding a single particle in the volume element $d\tau$, which is the absolute value of the statistical wave $\int \big|\psi^2\big|\, d\tau$.

This hydrodynamic model is however not adequate by itself for it contains nothing to describe the actual location of the particle: by examining a simple quantized state of a hydrogen atom, inspection shows that the guidance formula for the electron gives $\boldsymbol{v}=0$, which makes equation (1.26) irrelevant.

We may however circumvent this difficulty by considering a random perturbation of Brownian character superimposed onto the guided motion. In that case, the particle's regular motion obeying the guidance law should be subjected to a slight random influence of hidden origin, so as to switch from one guided trajectory to another.

The "main" trajectory would then appear as a "mean-valued motion".

Such a concept was brought forward by Bohm and Vigier [1] who referred to this invisible "thermostat" as the "sub-quantum medium". Referring to the same interpretation of the continuity equation (1.26), they showed that when random fluctuations would perturb the density ρ, a systematic tendency must exist for fluid elements to move to certain regions in such a way as to maintain the stability of the mean density ρ.

This tendency may find its origin in a kind of pressure which tends to correct the deviation.

A good example is a gas in a gravitational field in which the pressures automatically adjust themselves to maintain a local mean density close to

$$\rho = \rho_0 \exp\left(-\frac{mgz}{\mathfrak{K}T}\right),$$

where g is the acceleration of the force of gravity, and $\mathfrak{K}$ is Boltzmann's constant.

It should be stressed however, that the suggested medium does not serve as a universal reference system.

§2.2. The EGR picture

The velocity of light c will be taken here to be equal to 1.

When $V=0$ and $\boldsymbol{A}=0$, in a Riemannian situation, the relativistic continuity equation (1.15) may be conveniently generalized as

$$\left(g^{bc}\,\partial_b\,\phi\right)\partial_c\, a + \frac{1}{2}\, a\,\Box_{\text{Riem}}\,\phi = 0\,, \tag{2.1}$$

where $\Box_{\text{Riem}} = g^{bc}\,\nabla_b\nabla_c$.

In the framework of the Riemannian relativistic hydrodynamics, the classical continuity equation (1.26) for a neutral mass density ρ is

$$\nabla_a\left(\rho u^a\right) = 0\,, \tag{2.2}$$

where the unit velocity $u^a = \frac{dx^a}{ds}$ obeys

$$g_{ab}\,u^a u^b = g^{ab}\,u_a u_b = 1\,.$$

If we define the "guidance lines" by the differential equations (in the absence of external potentials)

$$g^{bc}\,\partial_c\,\phi = u^b,$$

where u^b generalizes the three-spatial guidance velocity $\boldsymbol{v}$ defined by the equation (1.19), which characterizes the flow lines of the fluid of proper density ρ

$$\mathbf{v} = v^\beta = \frac{u^\beta}{u^4}\,. \tag{2.3}$$

To maintain the form given by (1.26) it is easy to see that we must set

$$\rho = a^2 u^4 \tag{2.4}$$

and, taking into account (1.19), we obtain

$$\rho = a^2\left(-\,\partial_t\,\phi\right)\,, \tag{2.5}$$

with the space-time signature $(-+++)$.

To apply the generalized equation (2.2), we must start from the tensor

$$T_{ab} = a^2\,u_a u_b \tag{2.6}$$

and the equations

$$u^a\,\nabla_a\,u_b = 0\,, \tag{2.7}$$

which are a differential systems satisfied by the flow lines, which expresses that those lines are geodesics of the Riemannian metric ds^2.

Following now the above example of a pressurized gas, we consider a neutral perfect fluid whose well-known tensor is

$$T_{ab} = \left(a^2 + P\right)u_a u_b - P\,g_{ab} \tag{2.8}$$

with a prescribed equation of state $a^2 = f(P)$.

Equation (2.7) takes the form in a holonomic frame

$$\dot{u}_b = h_{ab}\,\partial^a U\,, \tag{2.9}$$

(here $h_{ab} = g_{ab} - u_a u_b$ is the projecting tensor) with

$$U = \int_{P_1}^{P_2} \frac{1}{a^2 + P}\,dP\,.$$

The quantity $\dot{u}_b$ represent the acceleration of the flow lines satisfying the differential system (2.9). Those flow lines are everywhere tangent to the four-unit vector u_c.

The differential system (2.9) is also written as

$$u^a \nabla_a u_b - \partial_a (U h^a_b) = 0\,. \tag{2.10}$$

In this case, the continuity equation becomes now

$$\nabla_b (a^2 u^b) - a^2 u^b \partial_b U = 0\,. \tag{2.11}$$

By doing so, our final aim is to put in evidence a "perturbed" density $\tilde{a}^2$, while keeping the usual classical form

$$\nabla_b (\tilde{a}^2 u^b) = 0\,. \tag{2.12}$$

A rigorous demonstration of Lichnérowicz [3] states, concerning the aforementioned flow lines, that:

> "... the flow lines satisfying the differential system* $\dot{u}_b = h_{ab}\,\partial^a U$ are geodesics of the metric
>
> $$ds^{2\prime} = e^{2U} ds^2$$
>
> conformal to the Einstein metric ds^2."

In other words, the presence of an internal pressure P readily induces a conformal factor (here e^{2U}), which is referred to as the *fluid index*.

Let us now introduce ∇' as the covariant derivative operator of the conformal metric $ds^{2\prime}$. We also define the "current vector" C^a of the considered fluid, whose components are

$$C^a = e^U u^a.$$

The current vector C' of $ds^{2\prime}$ has covariant components defined by

$$C'_a = C_a\,,$$

so as to remain unitary in the new metric

$$g^{ab\prime}\, C'_a C'_b = e^{-2U} g^{ab} (e^U u_a)(e^U u_b) = 1\,.$$

The contravariant components of the vector C' are

$$C^{a\prime} = g^{ab\prime}\, C_b = e^{-U} u^a.$$

*The equation $\dot{u}_b = h_{ab}\,\partial^a U$ quoted by Lichnérowicz is given by formula (2.9) of this paper. — P.M.

Inspection shows that these flow lines are geodesics of $ds^{2\prime}$, according to

$$C^{a\prime}\,\nabla'_a\,C^{b\prime} = 0\,, \tag{2.13}$$

which are fully equivalent to equations (2.10).

Likewise, the continuity equation (2.11)

$$\nabla_a\,(a^2\,u^a) - a^2\,u^a\,\partial_a U = 0$$

must coincide with the one describing the fluid density $\tilde{a}^2$

$$\nabla'_a\,(\tilde{a}^2\,C^{a\prime}) = 0\,, \tag{2.14}$$

which amounts to the recognition that the perturbation exerted on the fluid density $\tilde{a}^2$, i.e. the pressure P, is implicitly described by the conformal derivative ∇'

$$\left\{{}^{\,d}_{ab}\right\}' = \left\{{}^{\,d}_{ab}\right\} + \left(\delta^d_b\,\partial_a U + \delta^d_a\partial_b U - g_{ab}\,\partial^d U\right)\,.$$

However, a conformal metric is not suitable for describing the physical influence of an external medium which is defined in the initial ds^2.

This model has nevertheless an interesting virtue: following the same pattern, we will see that it is possible to build a plausible representation in the framework of the EGR theory.

§2.3. The influence of the metric fluctuations

In the extended formulation of General Relativity, the EGR theory [4], we may establish a continuity equation analogous to (2.14)

$$\mathrm{D}_a\,(\tilde{a}^2\,u^a_{\mathrm{EGR}}) = 0\,, \tag{2.15}$$

where the four-velocity u^a_{EGR} has the form defined in the EGR theory [4].

We suggest the following interpretation. The fluctuating density $\tilde{a}^2$ is related to the general connection defined in the EGR theory

$$\Gamma^d_{ab} = \left\{{}^{\,d}_{ab}\right\} + \left(\Gamma^d_{ab}\right)_J = \left\{{}^{\,d}_{ab}\right\} + \frac{1}{6}\left(\delta^d_a\,J_b + \delta^d_b\,J_a - 3\,g_{ab}\,J^d\right).$$

Unlike the conformal metric, which does not present a physical significance, the EGR theory provides a consistent scheme which enables to consider a one-to-one influence from an external medium manifestly represented by the "residual field" T^{ab}_{field}.

The "approximated" Riemannian continuity equation defined in the metric ds^2, which generalizes (2.12), should be written

$$\nabla_a\left(\langle\tilde{a}^2\rangle\,u^a\right) = 0\,. \tag{2.16}$$

The density a^2 in the Riemannian continuity equation would then appear as a *mean value* of the "instant" fluctuating density $\tilde{a}^2$ of the fluid, which actually obeys the equation (2.15) on the very small scale.

According to the EGR postulate, like for elementary particles, the Double Solution Theory is always considered in the framework of the dominant Riemannian geometry.

Bearing this in mind, remember that the wave ϑ is a *physical entity*, and so is the amplitude a, therefore the relativistic hydrodynamics applied here is fully legitimate.

Within the EGR theory, the Riemannian part of the "residual field" at its lowest level (but not vanishing) supplies the energy background ("thermostat") required by de Broglie's theory, and the small random disturbances are directly related to the covariant fluctuations of the metric through the non-Riemannian part of the persistent field.

Concluding remarks

As a concluding remark, let us stress that in this study we have made use of non-linear considerations, as we should in General Relativity, in accordance with de Broglie's ideas.

Francis Fer [5] has constructed a non-linear equation corresponding to the equation (1.13), and showed that in this general case, the relativistic continuity equation (2.1) defined in a Minkowski space remains unaffected. This remarkable result lends support to the aforementioned interpretation based on the EGR theory.

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