On the Evolution of the Fundamental Physical Constants

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Abstract: This is a presentation held, by Kyril P. Stanyukovich, on May 12, 1971, in Kiev, at the Institute of Theoretical Physics at the seminar on General Relativity maintained by Alexei Z. Petrov. Here Stanyukovich proposes his original theory of evolution of the fundamental physical constants with cosmological time, based on relations between the cosmological and quantum constants. He shows that, given only three experimentally measured fundamental physical quantities G, c, and $\hbar$, and also the scalar curvature R of space, which is changing with time, it is possible to express all rest-frame fundamental constants in terms of the aforementioned four basic parameters. Translated from the Russian manuscript of 1971 by Dmitri Rabounski, 2008. The translator thanks Andrew K. Stanyukovich, Russia, for permission to publish this paper, and also William C. Daywitt, USA, for assistance.

First we introduce the following definition of the gravitational mass, m_g, proceeding from the "linear quantum theory" authored by M. P. Bronstein [1]

$$m_g = \frac{\hbar}{ca}\,, \tag{1}$$

where a is the radius of the Metagalaxy. On the other hand, as one assumes,

$$\frac{m_g}{m_p} = \frac{2Gm_g^2}{\hbar c}\,, \tag{2}$$

where m_p is the mass of a nucleon. Because

$$m_p = \frac{\hbar}{\lambda c}\,, \tag{3}$$

where λ is the Compton wavelength of the nucleon (the "size" of the nucleon), we obtain, on the basis of the formulae (1), (2), and (3), that

$$\frac{m_g}{m_p} = \frac{\lambda}{a} = \frac{2G\hbar}{c^3\lambda^2} = \frac{L^2}{\lambda^2}\,,$$

where $L = \sqrt{2G\hbar/c^3}$ is the Planck length. Thus we obtain the fundamental relaton

$$\lambda^3 = L^2 a = \frac{2G\hbar a}{c^3} = \frac{2G\hbar}{c^2 H}\,, \tag{4}$$

where $H = \frac{c}{a}$ is Hubble's constant.

Let us check the validity of the resulting fundamental relation (4), and how this relation works. Substituting

$$G = 6.7\times10^{-8}\ \text{cm}^3/\text{g}\times\text{sec}^2\,, \qquad \hbar = 6.6\times10^{-27}\ \text{erg}\times\text{sec}\,,$$

$$c = 3\times10^{10}\ \text{cm/sec}\,, \qquad H = 10^{-17}\ \text{sec}^{-1},$$

into (4), we obtain $\lambda^3 \simeq 10^{-38}\,\text{cm}^3$ and $\lambda \simeq 10^{-13}\,\text{cm}$ that meets the real numerical values of these quantities.

We introduce one relation more, namely

$$\frac{2GM_0\,m_g}{\hbar c} = 1\,, \tag{5}$$

where M_0 is the mass of the Metagalaxy. Proceeding from (1) and (3), we have

$$\frac{2GM_0}{c^2} = a = r_{gM}\,, \tag{6}$$

where r_{gM} is the gravitational radius calculated for the entire Metagalaxy (the gravitational radius of the Metagalaxy). In actuality, at the present epoch the size of the Metagalaxy equals its gravitational radius. Therefore the relation (6) is wide used in cosmology.

Thus we verified again the fundamental relation (5). We are going to check it numerically. Because, according to the modern bounds, M_0 in the order of 10^{56} g, and m_g is in the order of 10^{-66} g, we obtain

$$\frac{\frac{8}{3}\times10^{-7}\,10^{56}\,10^{-66}}{3\times10^{10}\,10^{-27}} = \frac{8}{9}\,\frac{10^{-17}}{10^{-17}} \approx 1\,,$$

so the relation (5) has been completely verified. Since, according to the main cosmological assumption, space-time as a whole is homogeneous and isotropic (the assumption of homogeneous time is equivalent to the law of conservation of energy), the relations (5) and (6) should be true not only now, but for all time.

Assume M_0, c, and λ to be constants which remain unchanged with time. This supposition permits the possibility for introducing the main scales of length, time, and mass. In such a case, from (6) and (4), we have

$$G \sim a\,, \qquad G\hbar \sim a^{-1},$$

thus we obtain

$$\hbar \sim a^{-2}.$$

Because $R = \frac{12}{a^2}$ in a homogeneous 4-space, we have

$$G \sim R^{-1/2}, \qquad \hbar \sim R\,.$$

Thus we verified, again, our views according to which several fundamental "constants" are functions of the scalar curvature R in a space-time of variable curvature. In such a case,

$$H \sim R^{1/2}.$$

So, we have already reached a very good result according to which: given three experimentally measured physical quantities G, c, and $\hbar$, and also the scalar curvature of space, R, which changes with time, all rest-frame functions of the fundamental physical quantities can be expressed in terms of these 4 main parameters.

We have already calculated $\lambda = 2\times10^{-13}$ cm. Thus proceeding from (6) we obtain M_0. Since

$$a = \frac{c}{H} = 10^{28} \text{ cm}\,,$$

we obtain

$$M_0 = \frac{a c^2}{2G} = 10^{56} \text{ g}$$

which is exactly the known numerical value 10^{56} g mentioned above. Proceeding from these, we obtain the matter density of the Metagalaxy

$$\rho = \frac{3}{4}\,\frac{M_0}{\pi a^3} \simeq 10^{-29} \text{ g/cm}^3,$$

then we calculate the mass of a nucleon

$$m_p = \frac{\hbar}{\lambda c} \approx 10^{-24} \text{ g}\,,$$

and also the mass of a gravition

$$m_g = \frac{\hbar}{ca} \approx 10^{-64} \text{ g}\,.$$

Accordingly we calculate the number of nucleons in the Metagalaxy

$$N_p = \frac{M_0}{m_p} = \frac{a^2}{\lambda^2} = T_m^2 \approx 10^{80}$$

and the number of gravitons in the Metagalaxy

$$N_g = \frac{M_0}{m_g} = \frac{a^3}{\lambda^3} = T_m^3 \approx 10^{120}$$

where $T_m = \frac{a}{\lambda} = \frac{c}{\lambda H} = \frac{\omega}{H}$ is the Dirac dimensionless time.

Now we are going to determine the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137}\,.$$

As L. D. Landau has already shown [2], the electron's charge hidden from observation can be 137 times larger than its observed charge e. This results from the polarization of the vacuum. In [3], assuming the density of dipole charges to be $\delta = \frac{3}{4\pi r^3}$, we obtained that the total charge as

$$e^* = 4\pi e \int_{r_1}^{r_2} \delta r^2 \, dr = 3 e \ln \frac{r_2}{r_1} \,, \tag{7}$$

where r_1 is the "minimal radius" of the particle, while r_2 is its "external size". The size r_1 should be understood as the Planck length L, and also the planckeon model of elementary particles should be taken into account.

According to the planckeon model, there in the centre of each particle a planckeon is located — an Einstein micro-universe whose size is $L = \sqrt{2G\hbar/c^3} = 10^{-33}$ cm. A planckeon, due to its own fluctuations, ejects a part of its substance into outer space: this "atmosphere" surrounding planckeons is observed as elementary particles.

Thus we are lead to

$$e^* = 3 e \ln \frac{\lambda}{L} = e \ln \left(\frac{\lambda}{L}\right)^3 = e \ln \frac{a}{L} = e \ln 10^{60} \simeq 138\, e \,. \tag{8}$$

The efficiency of the charge — the quantity which characteres the interaction between e and e^* — is $e e^* = \hbar c$. Thus

$$\frac{1}{\alpha} = \frac{\hbar c}{e^2} = 1 + \ln \left(\frac{\lambda}{L}\right)^3 , \tag{9}$$

where $const = 1$ has been introduced for an ultimate case where $\lambda = L$ and $\alpha = 1$. So we obtain

$$e e^* = 137 e^2 = \hbar c = \frac{e^2}{\alpha} \,.$$

From Y. Nambu's empirical formula which characterizes the whole "spectrum" of elementary particles, along the "spectrum" the ratio between the mass m of any elementary particle and the mass of an electron, m_e, is given by the law

$$\frac{m}{m_e} = \frac{2n}{\alpha} \,, \tag{10}$$

where n is an integer specific to the particle. Thus we suggest that the "relative particle mass" changes according to logarithm of the space curvature.

Since

$$\left(\frac{\lambda}{L}\right)^3 = \frac{a}{L} = \left(\frac{a}{\lambda}\right)^{3/2},$$

we obtain

$$e^* = 3\,e\ln\frac{\lambda}{L} = \frac{3}{2}\,e\ln\frac{a}{\lambda} = e\ln\frac{a}{L}\,,$$

which allows us the opportunity to interpret $1/\alpha$ as either the logarithm of the probability for any particle of the Metagalaxy to be inside the volume, equal to the volume of this particle, or the entropy of a nucleon, calculated per one particle

$$s = -\,k\ln W = k\ln\left(\frac{a}{\lambda}\right)^3 = \frac{2\,k}{\alpha} = 274\,k\,,$$

where k is Boltzmann's constant.

Because we have derived above how G and $\hbar$ change with the curvature R, we easily calculate

$$m_p = \frac{\hbar}{\lambda c} \sim R\,, \quad m_g = \frac{\hbar}{ca} \sim R^{3/2}, \quad L = \sqrt{\frac{2G\hbar}{c^3}} \sim R^{-1/4},$$

$$\frac{a}{L} \sim R^{-3/4}, \qquad e^2 \sim \frac{R}{\ln R}\,, \qquad \omega = \frac{c}{\lambda} = const,$$

where ω is the frequency of strong interactions. The frequency of electromagnetic radiation and the radii of the "Bohr orbits" (the first "Bohr orbit", for example) are

$$\omega_\delta = \frac{m_e c^2}{\hbar}\left(\frac{e^2}{\hbar c}\right)^2, \qquad r_\delta = \frac{\hbar^2}{m_e e^2}\,,$$

where $m_e \simeq 10^{-27}$ g is the mass of the electron which changes logarithmically with time. Thus we obtain (here $ct = a$)

$$\omega_\delta = \frac{m_e c^2}{\hbar}\,\frac{1}{\left(1+3\ln\frac{\lambda}{L}\right)^2} = \frac{m_e c^2}{\hbar}\,\frac{1}{\left(1+\frac{3}{2}\ln\frac{ct}{\lambda}\right)^2}\,. \tag{11}$$

Let a source of light move away from us with a velocity $v = \frac{r}{t_n}$, and be currently located at a distance r from us. In such a case the observed frequence of the source is determied by the relation

$$\omega = \omega_0\sqrt{\frac{1-\frac{r}{ct_n}}{1+\frac{r}{ct_n}}}\left(\frac{\alpha_0}{\alpha_n}\right)^2,$$

where ω_0 and α_0 are the numerical values of ω and α at the moment of time t_0, while α_n is the numerical value of α at the moment t_n when the light beam was radiated. Here $ct_0 + r = ct_n$, where r is the distance between us and the source of the light as mentioned above.

Since

$$\frac{\alpha_0}{\alpha_n} = \frac{1 + \frac{3}{2}\ln\frac{ct_0}{\lambda}\left(1 + \frac{r}{ct_0}\right)}{1 + \frac{3}{2}\ln\frac{ct_0}{\lambda}}\,,$$

we finally obtain

$$\omega = \omega_0\sqrt{\frac{1 - \frac{r}{ct_0}}{1 + \frac{r}{ct_0}}}\left(\frac{1 + \frac{3}{2}\ln\frac{ct_0}{\lambda}\left(1 + \frac{r}{ct_0}\right)}{1 + \frac{3}{2}\ln\frac{ct_0}{\lambda}}\right)^2, \tag{12}$$

$$\omega = \omega_0\sqrt{\frac{1 - \frac{r}{ct_n - r}}{1 + \frac{r}{ct_n - r}}}\left(\frac{1 + \frac{3}{2}\ln\frac{ct_n}{\lambda}}{1 + \frac{3}{2}\ln\frac{ct_n}{\lambda}\left(1 - \frac{r}{ct_n}\right)}\right)^2. \tag{13}$$

Let $r = \alpha ct_0$. Then (12) takes the form

$$\omega = \omega_0\sqrt{\frac{1-\alpha}{1+\alpha}}\left(1 + \frac{\ln(1+\alpha)}{\frac{2}{3} + \ln\frac{ct_0}{\lambda}}\right)^2. \tag{14}$$

Because $\frac{ct_0}{\lambda} \gg 1$ at the present epoch, the correction to the Doppler effect is infinitesimal; so it can be neglected in the calculation.

On the other hand, at the initial moment of time, when $\frac{ct_0}{\lambda} \simeq 1$, the "ageing effect" was able to have a strong effect on the violet shift in the spectral lines. Most probably, the formulae (12) and (13) should be corrected "logarithmically", because $\frac{m_e}{m_p} \sim \alpha^{1.5}$. In such a case, the exponent in the formula will not be 2, but approximately 3.5 that does not no change the essence of the problem.

Now we calculate the primordial temperature. The initial temperature of the electromagnetic radiation is

$$T_0 = \frac{m_p c^2}{k}\,\frac{\bar{e}^2}{\hbar c}\,, \tag{15}$$

where $\frac{\bar{e}^2}{\hbar c}$ should be in order of $\frac{1}{100}$ (this is because, given at least $t = 1\,\text{sec}$, we have $T_m = 10^{23}$ and $\frac{\bar{e}^2}{\hbar c} = \frac{1}{80}$).

Let us calculate the change of the temperature of electromagnetic radiation with time according to the theory of evolution of the fundamental constants we have suggested here.

Because the total energy radiated by a blackbody per one cm^3 is $\varepsilon = \frac{4\sigma}{c}$, where $\sigma = \frac{\pi^2 k^4}{60\,\hbar^3 c^2}$ is the Stefan–Boltzmann constant, the pressure of the blackbody radiation $p = \frac{1}{3}\varepsilon$ is

$$p = \frac{4\sigma}{3c}\,T^4 = \frac{\pi^2}{45}\left(\frac{k}{\hbar c}\right)^3 k\left(T^0_{EM}\right)^4. \qquad (16)$$

Because $k\,T^0_0 = \hbar\omega$, where $T^0_0 = const$ (an isotermic process) is the temperature of a nucleon, we obtain

$$k \sim \hbar \sim T_m^2\,, \qquad \frac{k}{\hbar} = const\,, \qquad p \sim k\left(T^0_0\right)^4.$$

Initially the pressure is $p_0 \simeq k\left(T^0_0\right)^4$, so we obtain

$$\frac{p}{p_0} = \frac{k}{k_0}\left(\frac{T^0_{EM}}{T^0_0}\right)^4 = \frac{1}{T_m^2}\left(\frac{T^0_{EM}}{T^0_0}\right)^4. \qquad (17)$$

So far, $\frac{p}{p_0} = T_m^{-3}$ in the isotermic expansion of the Metagalaxy. Therefore, as we showed in [4, Part II, §7],

$$\frac{1}{T_m^2}\left(\frac{T^0_{EM}}{T^0_0}\right)^4 = \frac{1}{T_m^3}\,,$$

hence we obtain

$$\frac{T^0_{EM}}{T^0_0} = \frac{1}{T_m^{1/4}}\,. \qquad (18)$$

Finally, proceeding from (15) and (18), we obtain

$$T^0_{EM} = \frac{T^0_0}{T_m^{1/4}} = \frac{m_p c^2}{k\,T_m^{1/4}}\,\frac{\bar{e}^2}{\hbar c} \simeq \frac{10^{-24}\,10^{21}}{3\times 10^{10}\,10^{-16}\,10^2} \simeq 3^\circ\,\mathrm{K}\,,$$

that equals the measured temperature of primordial photons. Because p and the number of primordial photons in one cm^3, n_{EM}, is connected through the obvious relation

$$p = 2\,c\,\hbar N_p\, n_{EM}^{4/3} \sim T_m^{-3}\,,$$

hence we have

$$n_{EM} \sim T_m^{-3/4}$$

and, because the initial number of primordial photons is

$$n_{EM_0} = \left(\frac{10^{95}\,10^{21}}{2\times 3\times 10^{10}\;\,7\times 10^{-27}\,10^{80}}\right)^{3/4} = 10^{39}\ \mathrm{cm}^{-3},$$

where we used $p_0 = \rho_0 c^2$, $\rho_0 = \frac{3}{4}\frac{M_0}{\pi L^3}$, $M_0 = \frac{L c^2}{2G}$ which are valid at the initially moment of time ($a = L$), allowing us to use the numerical value $p_0 = \rho_0 c^2 = 10^{95}\,10^{21} = 10^{116}$.

Given the present epoch, we obtain

$$n_{EM} = 10^9 \text{ cm}^{-3}.$$

It is obvious, proceeding from the theory of evolution of the fundamental constants suggested here, that the density of primordial photons should equal the number of gravitons per unit volume.

Naturally, because

$$k_0\, T_0^0 = m_{g_0} c^2 = E_0 = M_0\, c^2,$$

where E_0 is the total initial energy, and, on the other hand,

$$E_0 = k_0\, T_0^0\, n_{EM_0} V_0 \ = k\, T_{EM}^0\, n_{EM} V\,,$$

we obtain $n_{EM_0} V_0 = 1$. From here, as $N_g = T_m^3$ and $V \sim T_m^3$, we obtain

$$n_{EM_0} = \frac{1}{V_0} = n_g = \frac{N_g}{V} = const\,.$$

Thus it is easy to see that

$$k\, T_{EM}^0\, n_{EM} = \frac{E_0}{V} = p_g\,,$$

where

$$p_g = \frac{E_0}{V} = \frac{2GM_0^2}{V^{4/3}} \simeq 10^{-7} \text{ dynes/cm}^2 = \text{erg/cm}^3.$$

The pressure produced by the intergalactic field equals, within the order of the numerical estimates, the density of the energy of the gravitational fields. Thus the energy of the primordial radiation is found to be related directly to the energy of gravitational fields.

The energy of a primordial particle is

$$E_{PM} = k\, T_{EM}^0 = 10^{-15} \text{ erg},$$

corresponding to a rest-mass $m_{EM} = 10^{-36}$ g.

The ratio

$$\frac{E_{PM}}{E_p} = \frac{10^{-15}}{10^{-3}} = 10^{-12}$$

corresponds to the ratio specific to forces the weak interactions. This fact cannot be an accident.

It is interesting to note that the relation

$$\frac{q_{c_1}^2}{e^2} = \frac{1}{T_m^{1/4}} = 10^{-10},$$

where $q_{c_1}^2 = 10^{-30}$ characterizes the weak interactions, can also not be an accident. However the reason for this relation is not clear yet.

That fact that the frequency $\approx 10^{12}\,\text{sec}^{-1}$, which is specific to the Lamb shift, corresponds to the energy of the order 10^{-15} erg can also not bound to be an accident.

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