On Increasing Entropy in an Infinite Universe

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Abstract: In this 1948 presentation Kyril P. Stanyukovich concludes that increasing the entropy of an infinite universe does not lead to a state of equilibrium, but only to a non-cyclic evolution of matter. A very truncated Russian version of this presentation was published in 1949 as: Stanyukovich K. P. O vozrastanii entropii v beskonechnoy vselennoy. *Doklady Akademii Nauk USSR*, 1949, vol. LXIX, no. 6, 793–796. Translated from the Russian manuscript of 1948 by Dmitri Rabounski, 2008. The translator thanks Andrew K. Stanyukovich, Russia, for permission to reproduce the original version of this paper, and also William C. Daywitt, USA, for assistance.

Relativistic thermodynamics shows that a universe does not approach equilibrium by increasing the universe's entropy [1]. In contrast, classical mechanics, statistical mechanics, and thermodynamics come to the opposite conclusion, or they leave the question unanswered.

One often assumes [2] either that unknown physical conditions in the universe lead to a decreasing entropy, or that fluctuations in the infinitude of regions of the universe lead to decreasing entropy as well, that compensate the thermodynamical processes of increasing entropy.

Modern statistical mechanics, which was developed after J. W. Gibbs through the studies of G. D. Birkhoff and A. J. Khinchin [3], considers very large (but finite) sets of particles. As a result, modern statistical mechanics gives no direct answer to the important principal question: is a universe with increasing entropy approaching a state of equilibrium in all its finite regions, or not?

Authors of numerous other studies naturally recognize that entropy increases in most cases of closed and finite systems, while statistical methods are often assumed to apply to an unbounded universe. Nevertheless, even though the infinite universe may be closed as a whole, statistical calculations do not apply to its entirety. In particular, it is wrong to claim that, given an increasing entropy, the universe will automatically approach a state of equilibrium [4]. J. I. Frenkel [5] noted that the entropy of a system, which interacts only minimally with the rest of the universe, increases with time due to the perturbing action of this interaction on the motion of particles in this system. According to Liouville's theorem, a completely isolated system has the property that a given volume $\triangle\Gamma$ of phase space remains unchanged with time. This result follows because the entropy $s \sim \ln \triangle\Gamma$.

Let me now develop my own view on the impossibility of an infinite universe reaching a state of equilibrium.

Split such an infinite universe into a countable (countably infinite) set of finite regions. Clearly, such a splitting is possible.

Obviously each finite region contains a finite number of elementary particles of matter. Because we want to take into account the interaction between these particles and any fields (electromagnetic and gravitational) that may be present, we assume with the quantum theory that a finite region of the universe contains a finite number of quanta. We also assume that the elementary quanta are infinitely small, not in the sense of "energetic points", but in the sense that the countable set of such quanta occupies a finite volume and contains a finite energy.

Therefore, a finite volume of space can contain not only a finite number of elementary particles (including quanta), but also a countable set of them.

In such a case, a countable set of elementary particles inhabits the entire space.

Clearly the set of interactions per a finite interval of time between the particles located in each finite volume of space forms a countable set of interactions if the particles in that set are countable, and forms a finite set if the number of particles is finite. Thus in both cases a countable set of interactions will be realized within the entire infinite space during a finite interval of time. The term "interaction" here means any process in which two particles exchange energy.

Because any infinite interval of time can be split into a countable set of finite intervals, a countable set of interactions can be realized in the entire universe during an infinite interval of time.

Classical statistics, when applied to an infinite universe, has the drawback that it assumes such a universe contains particles of only a single class (an unlikely situation in our Universe). It should also be noted that not all of the theorems of classical statistics are applicable to infinite sets of particles because those theorems only operate on finite sets. So applying these theorems to an infinite set of particles is not correct and can lead to untrustworthy results.

I suggest that, if an infinite universe were inhabited by a countable set of particles of the same class (e.g., like molecules), even in the case where each particle is in the same k energy level allowed to that particle, the universe would evolve to a state of equilibrium after a countable number of interactions between the particles (any and all types of particles are envisioned).

The above is obvious in the case of a finite number of energy levels, because the set of independent distributions of the particles in these levels is of the order of ω^{k-1} (here ω is the number of the particles). In the case of a countable set of levels, the corresponding set of independent distributions of the particles is also a countable set.

Thus we can suppose that, during a finite or infinite interval of time $t \leqslant \infty$, an infinite universe consisting of particles of the same class (excluding their gravitation fields) will arrive at a state of equilibrium. In the case of a countable set of energy levels, the state of equilibrium will also be reached at $t \leqslant \infty$ (this is due in part to the fact that each particle's energy is finite).

Let us introduce, as a postulate, the assumption that a countable set ($\Omega \to \infty$) of classes of different "particles" inhabit an infinite universe, where particles of a class Ω_i can consist of particles of "lower" classes Ω_{i-1}, Ω_{i-2}, ... We can envision such a "particle" as any autonomous structure such as a photon, a molecule, a star, or a stellar system, etc. We can also assume that such an infinite variety of classes of different particles is the result of an interaction between the structure and its fields. Any number of each type of particle can be present in the universe (clearly the number of each type can be infinite). Due to interactions within the countable set of particles of different classes, particles of the same classes and, perhaps, particles of new classes can be born. Given the aforementioned postulate, relations between particles of different classes are inexhaustible as are the results produced by those interactions. Of course, in the interactions of these particles, processes of "association" and "destruction" of other particle types can result. The assumption of strongly one-way processes, however, is not allowed as such an assumption would contradict the experimental evidence. It is enough that a countable set of particles of different classes be present, and that we assume for the particles of each class that the countable set of processes in the class is accompanied by at least one process of the opposite direction.

Considering particles of the same class, the equilibrium state of a system of these particles excludes all other states. The inevitable fluctuations in such a system, however, always lead the system to numerous "states of equilibrium" which differ from each other by a small value. I call such an equilibrium *absolute equilibrium*.

In the case of a countable set of classes of different particles, the term "absolute equilibrium" has no meaning. Naturally, according to the postulate, an infinite universe always contains several non-empty sets of particles of each class (we assume that these are countable sets of particles), and that the entire universe — the set of particles of all classes — is already in a state of equilibrium. We therefore consider the set of particles of a class Ω_i, for instance. Because particles of the lower classes Ω_{i-1}, Ω_{i-2}, ... are elements consisting of particles of the class Ω_i, interactions between particles of the class Ω_i can perturb particles of the lower classes. Therefore interactions between these particles will act on particles of the lower class Ω_{i-1} and also on particles of all other lower classes is such a way that systems of the lower-class particles will never be in an equilibrium state.

Because the order of a class i is unbounded, any structures in the universe can never be in a state of equilibrium. Thus the universe cannot approach a state of equilibrium. So a claim about a state of equilibrium for the entire universe should be looked upon with skepticism.

Clearly, absolute equilibrium can be reached in an infinite universe only if "particles" of different classes, which inhabit this universe, degenerate into "particles" of a single class. As shown above, however, this is not possible. Thus real interactions lead to such states, where substance of the universe experiences permanent evolution.

As interactions between particles of a class Ω_i cause particles of a lower classes to be in a non-equilibrium state, and as the number of particles in each class is variable, the clear result of these interactions will be a set of particles that approaches an equilibrium state. This follows because, as the set of particles reach new states again and again, these states are (more often than not) at a higher level of entropy than the previous states.

Because the order of a class i is unbounded, the result of these interactions leads to a "non-cyclic" evolution of matter that persists indefinitely.

It is interesting to note that the set of all formally imaginable distributions of particles of different classes among their respective energy levels acts as a continuum; so the number of possible sets is effectively inexhaustible.

So finally we arrive at the conclusion that an increasing entropy in the visible part of our Universe is not a factor in causing the Universe to approach its equilibrium state, but is a result of a permanent, non-cyclic, evolution of matter.

Discussion. When I suggested this theory (in the beginning of 1948), the core of which is my thesis that a countable set of molecules of the same class among other classes is always in a state of non-equilibrium, I met with some criticism from I. R. Plotkin. He told me that my conclusion proceeded from the erroneous belief that, given a countable set of particles, the set of independent distributions of the particles among their different states is also a countable set.* Here I would like to answer this criticism in detail.

1. Any infinite universe can be split into a countable set of regions.

2. Each finite region contains a finite number of particles, which is a countable set as well.

3. Thus, the entire space of an infinite universe contains a countable set of particles.

4. Thus, a countable set of interactions between the particles takes place in the entire space during a finite interval of time.

5. Any infinite interval of time can be split into a countable set of finite intervals. So, a countable set of interactions takes place in an infinite universe during infinite interval of time.

6. The set of all possible interactions in a countable set of particles is the set of all sub-sets of the countable set. This set has the power of continuum.

7. A countable set of interactions taking place during even an infinite interval of time cannot exhaust that continuum of interactions which are possible in the set.

8. Suppose an infinite universe is filled with particles of a single class. In such a case the set of all states the particles occupy is $N_n = k^n$ which is a continuum, where k represents a finite number of the energy levels, while $n \to \infty$ is the number of the particles. In the general case, the set of independent distributions of n particles among the energy levels is $N_k = \frac{(n+k-1)!}{n!(k-1)!}$. Having $n \to \infty$ (our case) gives $N_k = \frac{n^{k-1}}{(k-1)!}$, i.e. N_k is a *countable infinity* in our case. Points which characterize the non-equilibrium states in the phase space are distantly separated from the point of "absolute equilibrium" therein. The set of these points is only countable because the set of distributions of the particles among their energy levels is $\frac{n^{k-1}}{(k-1)!}$. Thus, if an infinite universe consists of a single class of particles, such a universe can reach a state of equilibrium only after a countable set of interactions among the particles has taken place, i.e. during an infinite amount of time, while all the rest "bank" of the continuum of the possible interactions was remained unused.

Consider the set $\bar{M}$ of all possible states for the infinite universe. Select the sub-set $\bar{n} \leqslant \bar{M}$ of these states where the universe is in a state

*Later Plotkin has published his criticism in a valuable Soviet journal of physics: Plotkin I. R. *JETP-USSR*, 1950, vol. 20, no. 11, 1051. — Editor's comment. D.R.

of equilibrium. Transitions inside of each "factor-set" $\frac{\bar{M}}{\bar{n}} \neq \bar{n}$ are due to increasing entropy, and are still evolving toward a state of equilibrium. Extract an element $\alpha\bar{n} \in \frac{\bar{M}}{\bar{n}}$ different from $\bar{n}$, i.e. $\alpha\bar{n} \prec \bar{n}$ (here $\prec$ means "much less than"). Assume that, at the moment of time $t = t_0$, the universe is in one of the states of the class $\alpha\bar{n}$. As such the universe will experience the transitions $\alpha\bar{n} \to \beta\bar{n} \to \gamma\bar{n} \to \ldots$ for which $\alpha\bar{n} \prec \beta\bar{n} \prec \gamma\bar{n} \prec \ldots$ We denote $\bar{A}$ as the power of the set of all the transitions experienced by the universe from $t = t_0$ until $t \to \infty$, while $\bar{B}$ denotes the power of the set of all transitions which are necessary for the universe to be in the states of the class $\bar{n}$ (i.e. to be in the state of equilibrium). The universe consisting of particles of the same class is always in a state of non-equilibrium if and only if $\bar{A} < \bar{B}$. However, the opposite condition $\bar{B} < \bar{A}$ is true for the two obvious reasons: 1) given a countable set of particles of the same class, the set of their independent distributions among their energy levels is a countable set; 2) considering heat-conduction or diffusion of a gas in an unbounded space, we conclude that, even if an extremely lopsided distribution of heat exists in the space (where all heat has been condensed into a small region in which the energy density is infinite e.g.), heat eventially becomes equally distributed in the space after an infinite amount of time, so the state of the gas becomes with time only infinitesimally different from the equilibrium state.

9. Imagine an infinite universe filled with a countable set $\Omega \to \infty$ of classes of particles, where each particle of a class Ω_i can contain particles of all lower classes (Ω_{i-1}, Ω_{i-2}, ...). As the assumption of only one-way processes is unacceptable, there are processes of both association and dissociation of the particles. Once a single process appears among a set of exclusively opposite processes in the same class of particles, a countable set of both classes of particles will be generated from the original set some time later.

10. In such a case, not only the set of all possible states, but also the set of all independent dispositions of the different particles among their energy levels, will exist; leading to

$$N_{km} = \prod_{j=1}^{j=m\to\infty} \frac{(n_j + k - 1)!}{n_j (k-1)!} \longrightarrow \left(\frac{n^{k-1}}{(k-1)!}\right)^m \sim 2^m,$$

where $m \to \infty$ is the number of particle classes.

11. Thus, despite increasing entropy in each finite region of the infinite universe, the entire universe containing the countable set of different particle classes is always in a state of non-equilibrium and is unable to reach equilibrium.

12. So, a countable set of particles of the same class reaches the state of equilibrium only through *regular infinity* (i.e. *actual infinity*) after a countable set of interactions among the particles has taken place (see Thesis 8). Therefore Plotkin was wrong when criticized my thesis that a countable set of molecules of the same class among other classes is always in a state of non-equilibrium. He was wrong as well when claiming that a permanent strong non-equilibrium state is specific to a countable set of particles of the same class, if this is the single class of particles in the universe. On the contrary, the universe is able to be in a non-equilibrium state due to the many-level internal structure of the particles which inhabit it.

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*Sergei Ivanovich Vavilov (1891–1951), a Russian research physicist who was the President of the USSR Academy of Sciences in the years 1945–1951. — Editor's comment. D.R.

†Nikolai Nikolaevich Bogoliubov (1909–1992), a Russian mathematician and theoretical physicist, and a member of the USSR Academy of Sciences. — Editor's comment. D.R.

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